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THE SOFTWARE ELECTRE III-IV

METHODOLOGY AND USER MANUAL (VERSION 3.X)

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PARIS, FRANCE

September, 2006

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Preamble

This document has two main objectives: to make a brief presentation of ELECTRE III, and ELECTRE IV methods; and to help the user of the ELECTRE III-IV software. It is based on two manuals in French elaborated by Dominique VALLÉE and Pitor ZIELNIEWICZ [67, 68].

ELECTRE III-IV software, version 3.x, was developed with Borland C++ programming language using the Microsoft Windows interface, by Pitor ZIELNIEWICZ (Institute of Computing Science of the Poznan University of Technology) under the supervision of Professors Bernard ROY and Roman SŁOWIŃSKI.

For more questions about ELECTRE III-IV software, please ask: Professor Bernard ROY; LAMSADE - Université Paris-Dauphine; Place du Maréchal De Lattre de Tassigny; 75775 Paris Cedex 16 - France; email: `roy@lamsade.dauphine.fr`; Tel: (+33 1) 44 05 42 87; Fax: (+33 1) 44 05 40 91.

Chapter 1 presents the theoretical foundations of ELECTRE III and ELECTRE IV methods For a detailed presentation of these methods consult [48]. Chapter 2 presents the ELECTRE III-IV software, with the help of the interface, such as the ones corresponding to the input data, the calculation, and the interpretation of the results. The bibliography presents the main references, and applications of the two methods; and, finally, the index presents the methodology concepts, and the commands of the software.

This work has benefited from the Luso-French cooperation under *Programa PESSOA*. The first author was also supported by the grant SFRH/BD/22985/2005 (Foundation of Science and Technology, Portugal)

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Chapter 1

The ELECTRE Methodology for Ranking Problems

ELECTRE methods comprise two phases: construction of one or several outranking relation(s) followed by an exploitation procedure. ELECTRE III and ELECTRE IV methods aim to answer the following question: considering a finite set of actions, A, evaluated on a coherent family of pseudo-criteria, F, how to make a partition of A in classes of equivalence and provide a necessarily complete pre-order expressing the relative position of these classes? In the first phase, ELECTRE III method involves the construction of a fuzzy outranking relation and ELECTRE IV the construction of a set of a nested sequence of outranking relations. In the second phase, an algorithm is used for making a ranking in a final partial pre-order, that combines two complete pre-orders.

Consider a set of actions evaluated on several pseudo-criteria; there are three different cases that should be taken into account, in order to build the outranking relations:

- The decision-maker is able to express the relative importance of the pseudo-criteria (use of ELECTRE III method).

- The decision-maker is not able, does not want, or cannot express the relative importance of each criterion (use of ELECTRE IV method). However, using ELECTRE IV is only valid if the following two conditions are satisfied: no criterion is either preponderant or negligible when compared to any subset of half of the criteria.

- The decision-maker has already a pairwise comparison matrix of the actions that has been obtained by a different method (neither ELECTRE III nor ELECTRE IV). The elements of this matrix are within the range [0,1] as it is the case of the fuzzy outranking relation of ELECTRE III. These values are used to run the ranking algorithm (use of *Matrix of degrees de credibility* type project).

1.1 The ELECTRE III method

ELECTRE III method starts by a pairwise comparison of each action[1] to the remaining ones with the aim of accepting, rejecting, or, more generally, assessing the credibility of the assertion "action a is at least as good as action b", usually called "a outranks b" (denoted $a\,S\,b$) taking into account the following three aspects:

- the indifference and preference thresholds defined for each criterion.
- the degree or coefficients of importance attached to each criterion.
- the possible difficulties of relative comparison of two actions when one is significantly better than the other on a subset of criteria, but much worst on at least one criterion from a complementary subset.

For each criterion, two indices should be calculated. One expresses in what measure the performances of the actions a and b are in concordance with the assertion "a outranks b"; the other indicates in what measure they oppose this assertion. For such, the partial concordance indices are aggregated while taking into account the relative importance of the criteria to give birth to the comprehensive concordance indices (it should be notice that the partial discordance indices are not aggregated). The fuzzy outranking relations, defined for each pair of actions (a,b) as a credibility index, $\sigma(a,b)$, express comprehensively in what measure "a outranks b" using both the comprehensive concordance index and the discordance indices for each criterion g_j. By applying the ranking algorithm and using the distillation threshold, the final results provide a partial pre-order (see Figure 1.1).

[1] Sometimes *alternative* is used instead of *action*. It is not a semantic question, because the concept of *alternative* corresponds to the particular case in which modeling is such that two distinct action cannot be conjointly put into operation [46, p. 8]

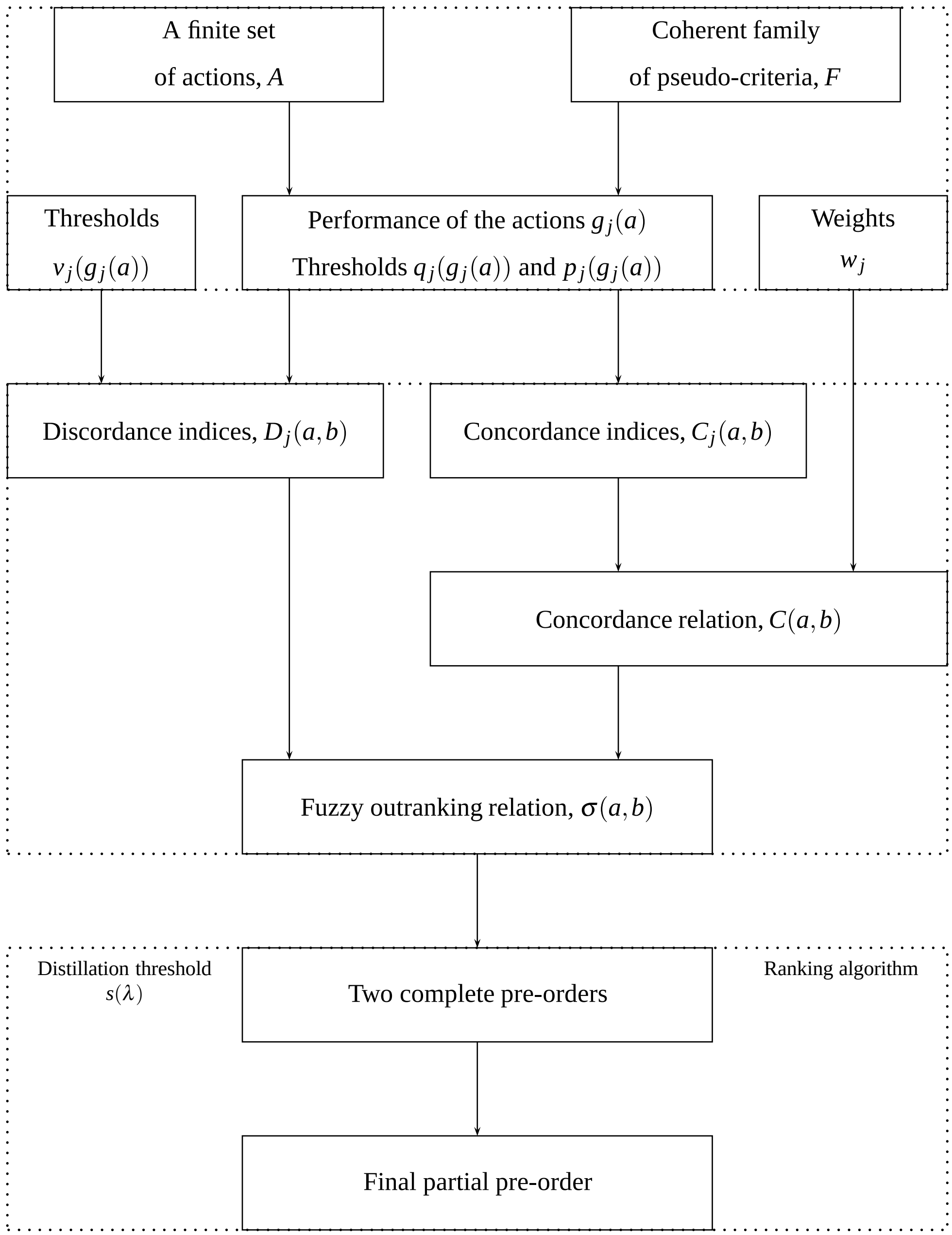


Figure 1.1: General structure of ELECTRE III

1.1.1 Notation

A $= \{a_1, a_2, \dots, a_i, \dots, a_m\}$: a finite set of actions to rank, with $|A| = m$;

F $= \{g_1, g_2, \dots, g_j, \dots, g_n\}$: a finite set of coherent family of criteria, with $|F| = n$;

$g_j(a)$: the evaluation (or performance) of action a on criterion g_j, for $j = 1, 2, \dots, n$;

w_j : the relative importance coefficient attached to criterion g_j, for $j = 1, 2, \dots, n$;

$q_j(g_j(a)) = \alpha_j \times g_j(a) + \beta_j$: the direct indifference threshold of action a compared with action b, when the preferences are in the increasing direction and $g_j(a) < g_j(b)$, for $j = 1, 2, \ldots, n$;

$p_j(g_j(a)) = \alpha_j \times g_j(a) + \beta_j$: the direct preference threshold of action a compared with action b, when when the preferences are in the increasing direction and $g_j(a) < g_j(b)$, for $j = 1, 2, \ldots, n$;

$v_j(g_j(a)) = \alpha_j \times g_j(a) + \beta_j$: the direct veto threshold of action a compared with action b, when the preferences are in the increasing direction and $g_j(a) < g_j(b)$, for $j = 1, 2, \ldots, n$;

I_j : the indifference relation on criterion g_j;

P_j : the strict preference relation on criterion g_j;

Q_j : the weak preference relation on criterion g_j (it means an hesitation between indifference and strict preference);

PV_j : the veto-preference relation on criterion g_j.

1.1.2 The thresholds

There are different criteria models. In the true-criterion model, the smallest difference in performances between two actions, a and b, leads to a strict preference for one of the two actions in the comparison with the other:

$a\, P_j\, b \Leftrightarrow g_j(a) > g_j(b)$: it means that a is strictly preferred to b on criterion g_j.

$a\, I_j\, b \Leftrightarrow g_j(a) = g_j(b)$: it means that a is indifferent to b on criterion g_j.

The pseudo-criterion model allows, with the use of thresholds, to take into account the ill-determination, imprecision, and uncertainty that may affect performances. For instance, in the case of increasing preference direction, let $q(.)$ and $p(.)$ be the indifference and preference thresholds, respectively:

- an action b such that $g(b)$ is greater than $g(a)$ but smaller than $g(a) + q(.)$ will be considered indifferent to a.

- an action b such that $g(b)$ is greater than $g(a) + p(.)$ will be considered as strictly preferred to a.
- an action b such that $g(b)$ is greater than $g(a) + q(.)$ but smaller than $g(a) + p(.)$, the preference will be considered as not significantly established.

The comparison of actions in the way that has just been described before leads to the construction of a concordance index for each pair of actions (a,b), which expresses to what extent the criterion is in harmony with the assertion "a is at least as good as b".

Moreover, ELECTRE methods allow to introduce the notion of veto: it is said that a criterion vetoes the validation of the assertion "action a is at least as good as b" if the difference of performances is so important in favor of b that it prevents the possibility that, comprehensively, action a should be considered as at least as good as action b. The difference that leads to the notion of discordance is called veto threshold.

The three thresholds can be defined as follows:

- the **indifference threshold** corresponds to the largest difference of performances between two actions, compatible with an indifference situation;
- the **preference threshold** corresponds to the smallest difference of performances between two actions from which the decision-maker strictly prefers the action presenting the best performance;
- the **veto threshold** is the smallest difference of the performances between two actions from which the decision-maker considers that it is not possible to support the idea that the worst of the two actions under consideration on a certain criterion may be comprehensively considered as good as the better one, even if its performances on all the other criteria are better.

Consider a and b two actions to be compared: the difference in their performances should be compared, firstly with the indifference threshold, then with the preference threshold, and lastly with the veto threshold.

To avoid some incoherences, two conditions are associated to the pseudo-criteria model and a third one to the veto threshold [42, 45, chap. 9]:

$$\frac{q_j(g_j(b)) - q_j(g_j(a))}{g_j(b) - g_j(a)} \geq -1 \tag{1.1}$$

$$\frac{p_j(g_j(b)) - p_j(g_j(a))}{g_j(b) - g_j(a)} \geq -1 \tag{1.2}$$

$$\frac{v_j(g_j(b)) - v_j(g_j(a))}{g_j(b) - g_j(a)} \geq -1 \tag{1.3}$$

The ELECTRE methods allow to use these thresholds on every or only some criteria. The perception of these thresholds may vary along the scale of performances. Moreover, it is possible to think of thresholds in terms of the worst performances of the two actions compared or in terms of the best one (in the first case the calculation of the thresholds is said to be direct, while in the second case it is said inverse).

The worst performance is the smallest one if preference direction is increasing and the greatest one if the preference direction is decreasing. Therefore, if $g(a)$ is smaller than $g(b)$ and if the preference direction is increasing, then the direct indifference threshold will be calculated with the formula: $\alpha \times g(a) + \beta$ and the inverse threshold with the formula $\alpha \times g(b) + \beta$. Keeping $g(a)$ smaller than $g(b)$, but with a decreasing preference direction, the direct indifference threshold to compare a and b will be calculated with the formula: $\alpha \times g(b) + \beta$, and the inverse one with $\alpha \times g(a) + \beta$.

The calculation of the thresholds can be made according to four different contexts because the direction of preference can be increasing or decreasing and the thresholds can be direct or inverse with respect to the performance. Therefore:

- **Case 1**: increasing preferences with respect to the performance and direct thresholds
- **Case 2**: decreasing preferences with respect to the performance and direct thresholds
- **Case 3**: increasing preferences with respect to the performance and inverse thresholds
- **Case 4**: decreasing preferences with respect to the performance and inverse thresholds

ELECTRE III-IV software allows the definition of each threshold as an affine function of the performance. For an action a with performance $g(a)$, the indifference, preference and veto thresholds are calculated in the following manner: $\alpha \times g(a) + \beta$. When the coefficient α is different to 0, the threshold varies as a function of the performance.

It is a decision-maker task to specify, for each criterion and for each type of threshold, the value of the coefficients α and β. For **consistency reasons** the coefficient α must be greater than or equal to -1 in the case of increasing preferences direction and the calculation of the threshold is direct (Case 1). In the same way, the coefficient α must be lower than 1 in the cases 2 and 3. As for the case 4, it must be greater than -1. In any case, the coefficients α and β must not return a negative value for a threshold. Besides, along the scale of a criterion, the indifference threshold must remain lower than the preference threshold itself lower than the veto threshold, if it exists, $q_j(.) \leq p_j(.) \leq v_j(.)$.

Case 1: Increasing preferences and direct thresholds

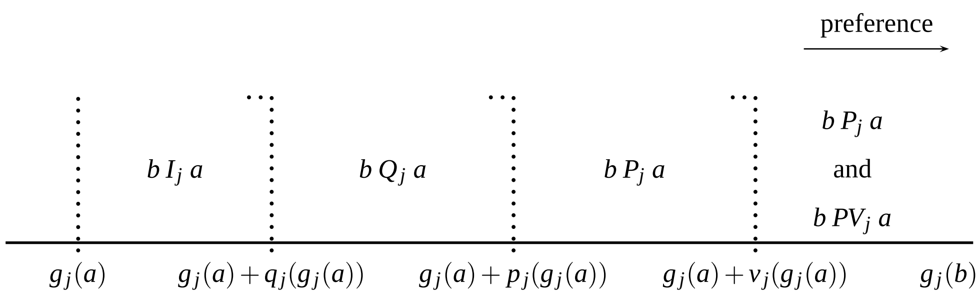


Figure 1.2: Calculation of the thresholds (Case 1)

Case 2: Decreasing preferences and direct thresholds

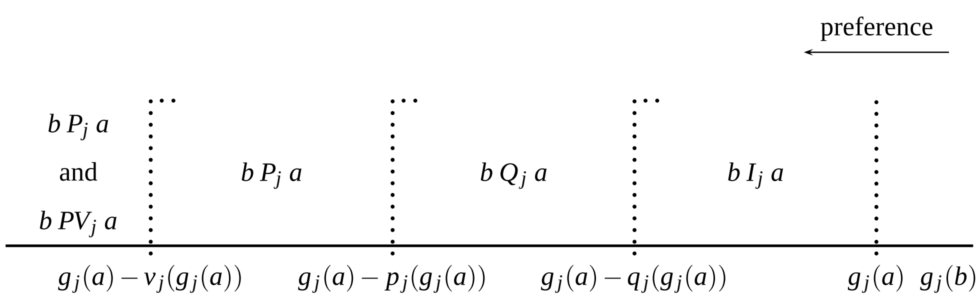


Figure 1.3: Calculation of the thresholds (Case 2)

For the pseudo-criteria model with veto thresholds, if $g_j(b) \geq g_j(a)$, the following relations occur:

- $b\ I_j\ a \Leftrightarrow g_j(b) - g_j(a) \leq q_j(g_j(a))$

- $b\ Q_j\ a \Leftrightarrow q_j(g_j(a)) < g_j(b) - g_j(a) \leq p_j(g_j(a))$
- $b\ P_j\ a \Leftrightarrow p_j(g_j(a)) < g_j(b) - g_j(a) \leq v_j(g_j(a))$
- $b\ PV_j\ a \Leftrightarrow g_j(b) - g_j(a) > v_j(g_j(a))$

Case 3 and 4: Inverse thresholds

In cases 3 and 4 the thresholds are inverse. ELECTRE III-IV software transforms the inverse thresholds automatically into direct ones, based in the principle of preferential preservation situations. It permits to use the same pairwise comparison algorithm whatever the type of thresholds considered. When in presence of case 3, the calculation can be made like in case 1, and in presence of case 4, the calculation can be made like in case 2, through the following transformations, in order to obtain direct coefficients of the thresholds functions (α_p, β_p, α_q, and β_q):

- $\alpha_p = \dfrac{\alpha_p^{'}}{1+\alpha_p^{'}};\ \beta_p = \dfrac{\beta_p^{'}}{1+\alpha_p^{'}}$
- $\alpha_q = \dfrac{\alpha_q^{'}}{1+\alpha_q^{'}};\ \beta_q = \dfrac{\beta_q^{'}}{1+\alpha_q^{'}}$

where, $\alpha_p^{'}$ and $\beta_p^{'}$ are the inverse preference thresholds coefficients and $\alpha_q^{'}$ and $\beta_q^{'}$ are the inverse indifference thresholds coefficients defined by the user. For more details on the transformations of the inverse coefficients into direct ones, consult [67, p. 14-16].

The values of the thresholds are used in the calculation of the concordance indices, the comprehensive concordance indices and the discordances indices, that are analyzed in the next three sections.

1.1.3 The concordance indices

Let (a,b) be a pair of actions, the concordance index, $C_j(a,b)$, is a fuzzy index measuring whether "action a is at least as good as action b" on criterion g_j. The calculation can be made according to the four different contexts that have been used for the definition of the thresholds. See an example of the calculation on page 25.

Case 1: Increasing preferences and direct thresholds

For a fixed $g_j(a)$, Figure 1.4 represents the variations of $C_j(a,b)$ according to the variations of $g_j(b)$. The thresholds are direct, therefore calculated from the least favorite value.

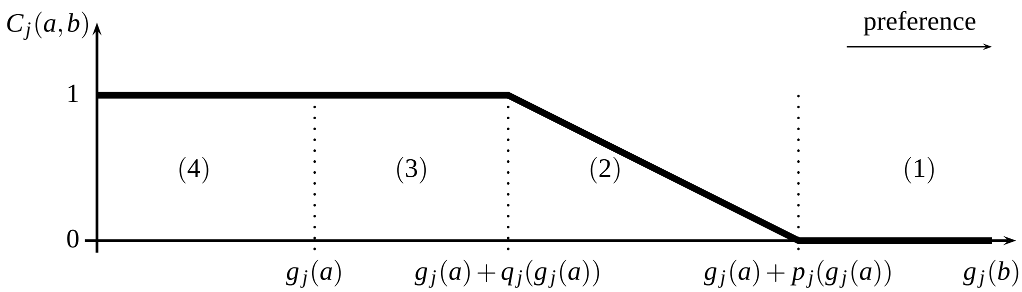


Figure 1.4: Partial concordance indices in ELECTRE III (Case 1)

In other words, if $g_j(b)$ belongs to the zone:

(1) : $g_j(b) > g_j(a) + p_j(g_j(a)) \Rightarrow C_j(a,b) = 0$

- . b is strictly preferred to a on criterion g_j.
- . Remark: $C_j(b,a) = 1$

(2) : $g_j(a) + q_j(g_j(a)) < g_j(b) < g_j(a) + p_j(g_j(a)) \Rightarrow 0 < C_j(a,b) < 1$

- . b is weakly preferred to a on criterion g_j.
- . Remark: $C_j(b,a) = 1$
- . $C_j(a,b) = \dfrac{p_j(g_j(a)) - [g_j(b) - g_j(a)]}{p_j(g_j(a)) - q_j(g_j(a))}$

(3) : $g_j(a) \leq g_j(b) \leq g_j(a) + q_j(g_j(a)) \Rightarrow C_j(a,b) = 1$

- . b and a are indifferent on criterion g_j.
- . Remark: $C_j(b,a) = 1$

(4) : $g_j(b) \leq g_j(a) \Rightarrow C_j(a,b) = 1$

- . a can be indifferent, weakly, or strictly preferred to b. In all the cases, the performance of a is better than the performance of b.

. Remark: the value of $C_j(b,a)$ depends on the $g_j(a)-g_j(b)$, $q_j(g_j(b))$, and / or $p_j(g_j(b))$.

The general formula of the calculation of the partial concordance index is, therefore:

$$C_j(a,b) = \frac{p_j(g_j(a)) - \min\left\{ \left[g_j(b)-g_j(a)\right], p_j(g_j(a)) \right\}}{p_j(g_j(a)) - \min\left\{ q_j(g_j(a)), \left[g_j(b)-g_j(a)\right] \right\}} \tag{1.4}$$

For the pair (b,a):

- $g_j(a) - q_j(g_j(b)) \leq g_j(b) \Rightarrow C_j(b,a) = 1$ (b and a are indifferent on criterion g_j).
- $g_j(a) - p_j(g_j(b)) \leq g_j(b) < g_j(a) - q_j(g_j(b)) \Rightarrow 0 < C_j(a,b) < 1$ (a is weakly preferred to b on criterion g_j).
- $g_j(b) < g_j(a) - p_j(g_j(b)) \Rightarrow C_j(b,a) = 0$ (a is strictly preferred to b on criterion g_j).

Case 2: Decreasing preferences and direct thresholds

For a fixed $g_j(a)$, Figure 1.5 represents the variations of $C_j(a,b)$ according to the variations of $g_j(b)$. The thresholds are direct, therefore calculated from the least favorite value.

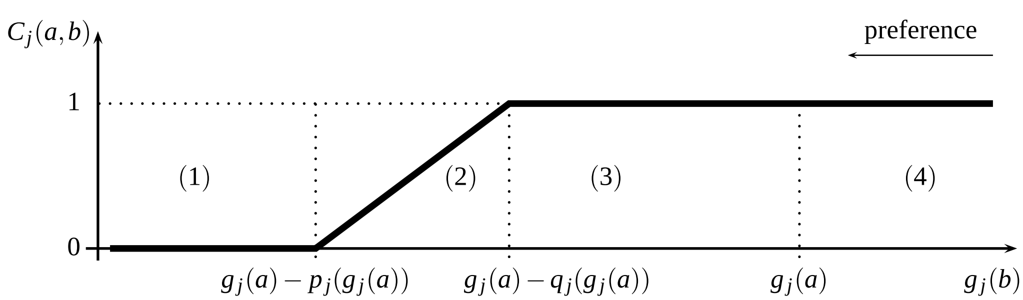


Figure 1.5: Partial concordance indices in ELECTRE III (Case 2)

In other words, if $g_j(b)$ belongs to the zone:

(1) : $g_j(a) - g_j(b) > p_j(g_j(a)) \Rightarrow C_j(a,b) = 0$

. b is strictly preferred to a on criterion g_j.

(2) : $q_j(g_j(a)) < g_j(a) - g_j(b) < p_j(g_j(a)) \Rightarrow 0 < C_j(a,b) < 1$

. b is weakly preferred to a on criterion g_j.

. $C_j(a,b) = \dfrac{p_j(g_j(a)) - \left[g_j(a) - g_j(b)\right]}{p_j(g_j(a)) - q_j(g_j(a))}$

(3) : $g_j(a) - g_j(b) \leq q_j(g_j(a)) \Rightarrow C_j(a,b) = 1$

. b and a are indifferent on criterion g_j.

(4) : $g_j(a) \leq g_j(b) \Rightarrow C_j(a,b) = 1$

. a can be indifferent, weakly, or strictly preferred to b. In all the cases, the performance of a is better than the performance of b.

Case 3 and 4: Inverse thresholds

In presence of case 3, the calculation can be made like in case 1, and in presence of case 4, the calculation can be made like in case 2, through the following transformations, in order to obtain direct coefficients of the thresholds functions (α_p, β_p, α_q, and β_q):

- $\alpha_p = \dfrac{\alpha_p^{'}}{1+\alpha_p^{'}}$; $\beta_p = \dfrac{\beta_p^{'}}{1+\alpha_p^{'}}$
- $\alpha_q = \dfrac{\alpha_q^{'}}{1+\alpha_q^{'}}$; $\beta_q = \dfrac{\beta_q^{'}}{1+\alpha_q^{'}}$

where, $\alpha_p^{'}$ and $\beta_p^{'}$ are the inverse preference thresholds coefficients and $\alpha_q^{'}$ and $\beta_q^{'}$ are the inverse indifference thresholds coefficients defined by the user.

1.1.4 The comprehensive concordance indices

The comprehensive concordance index, $C(a,b)$, is the sum of the partial concordance indices, $C_j(a,b)$, on each criterion weighted by the weights of each criterion, w_j. The value of

$C(a,b)$ express in what measure the performances on all criteria are in concordance with the assertion "*a* outranks *b*" (expression 1.5). See an example of the calculation on page 28.

$$C(a,b) = \frac{\sum_{j=1}^{n} w_j \, C_j(a,b)}{\sum_{j=1}^{n} w_j} \tag{1.5}$$

1.1.5 The discordance indices

The discordance of a criterion g_j aims to take into account the fact that this criterion is more or less discordant with the assertion "*a* outranks *b*'". The discordance index, D_j, reaches its maximal value when the criterion g_j puts its veto to the outranking relation; it is minimal when the criterion g_j is not discordant with the relation. For the calculation of $D_j(a,b)$, it is necessary to define a veto threshold for each criterion. See an example of the calculation on page 30.

Case 1: Increasing preferences and direct thresholds

For the first case, $D_j(a,b)$ can be defined, in a general way, by the following formula:

$$D_j(a,b) = \min\left\{1,\ \max\left\{0,\ \frac{\left[g_j(b) - g_(a)\right] - p_j(g_j(a))}{v_j(g_j(a)) - p_j(g_j(a))}\right\}\right\} \tag{1.6}$$

In Figure 1.6, if $g_j(b)$ belongs to the zone:

(1) : $g_j(b) - g_j(a) \leq p_j(g_j(a)) \Rightarrow D_j(a,b) = 0$

. The performances $g_j(a)$ and $g_j(b)$ on criterion g_j do not reject the assertion "*a* outranks *b*".

(2) : $p_j(g_j(a)) < g_j(b) - g_j(a) < v_j(g_j(a)) \Rightarrow 0 < D_j(a,b) < 1$

. The performances $g_j(a)$ and $g_j(b)$ on criterion g_j weakly reject the assertion "*a* outranks *b*".

. $D_j(a,b) = \dfrac{\left[g_j(b) - g_j(a)\right] - p_j(g_j(a))}{v_j(g_j(a)) - p_j(g_j(a))}$

(3) : $g_j(b) - g_j(a) \geq v_j(g_j(a)) \Rightarrow D_j(a,b) = 1$

. The performances $g_j(a)$ and $g_j(b)$ on criterion g_j reject the assertion "*a* outranks *b*".

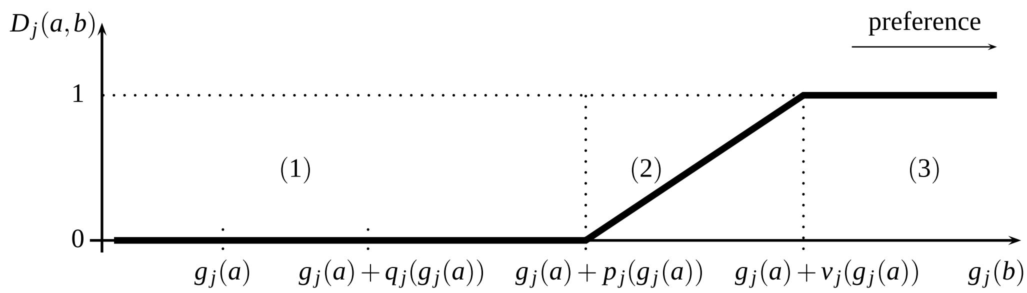


Figure 1.6: Partial discordance indices in ELECTRE III (Case 1)

Case 2: Decreasing preferences and direct thresholds

For the second case, $D_j(a,b)$ can be defined, in a general way, by the following formula:

$$D_j(a,b) = \min\left\{1,\ \max\left\{0,\ \frac{[g_j(a) - g_(b)] - p_j(g_j(a))}{v_j(g_j(a)) - p_j(g_j(a))}\right\}\right\} \tag{1.7}$$

In Figure 1.7, if $g_j(b)$ belongs to the zone:

(1) : $g_j(a) - g_j(b) \leq p_j(g_j(a)) \Rightarrow D_j(a,b) = 0$

. The performances $g_j(a)$ and $g_j(b)$ on criterion g_j do not reject the assertion "*a* outranks *b*".

(2) : $p_j(g_j(a)) < g_j(a) - g_j(b) < v_j(g_j(a)) \Rightarrow 0 < D_j(a,b) < 1$

. The performances $g_j(a)$ and $g_j(b)$ on criterion g_j weakly reject the assertion "*a* outranks *b*".

. $D_j(a,b) = \dfrac{[g_j(a) - g_j(b)] - p_j(g_j(a))}{v_j(g_j(a)) - p_j(g_j(a))}$

(3) : $g_j(a) - g_j(b) \geq v_j(g_j(a)) \Rightarrow D_j(a,b) = 1$

. The performances $g_j(a)$ and $g_j(b)$ on criterion g_j reject the assertion "*a* outranks *b*".

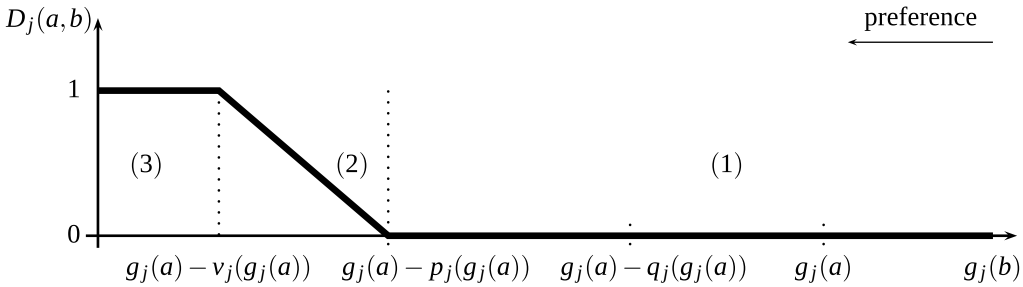


Figure 1.7: Partial discordance indices in ELECTRE III (Case 2)

Case 3 and 4: Inverse thresholds

In presence of case 3, the calculation can be made like in case 1, and in presence of case 4, the calculation can be made like in case 2, through the following transformations, in order to obtain direct coefficients of the thresholds functions (α_p, β_p, α_v, and β_v):

- $\alpha_p = \dfrac{\alpha_p^{'}}{1+\alpha_p^{'}}$; $\beta_p = \dfrac{\beta_p^{'}}{1+\alpha_p^{'}}$
- $\alpha_v = \dfrac{\alpha_v^{'}}{1+\alpha_v^{'}}$; $\beta_v = \dfrac{\beta_v^{'}}{1+\alpha_v^{'}}$

where, $\alpha_p^{'}$ and $\beta_p^{'}$ are the inverse preference thresholds coefficients and $\alpha_v^{'}$ and $\beta_v^{'}$ are the inverse veto thresholds coefficients defined by the user.

1.1.6 The fuzzy outranking relations in ELECTRE III

The fuzzy outranking relation, defined for each pair of actions (a,b) as a credibility index, $\sigma(a,b)$, express comprehensively in what measure "*a* outranks *b*" using both the comprehensive concordance index and the discordance indices for each criterion g_j. The credibility is merely the comprehensive concordance index weakened by the discordance indices. In the absence of such discordant criteria, $\sigma(a,b) = C(a,b)$. This credibility value is reduced in the presence of one or more discordant criteria when $D_j(a,b) > C(a,b)$. In conformity with the veto effect, $\sigma(a,b) = 0$ if $\exists\ j \mid D_j(a,b) = 1$, whatever the relative importance of the criterion, w_j.

The credibility index can be defined as follows:

$$\sigma(a,b) = \begin{cases} C(a,b) & \text{if } \overline{F}(a,b) = \emptyset \\ C(a,b) \times \prod\limits_{j \in \overline{F}(a,b)} \dfrac{1 - D_j(a,b)}{1 - C(a,b)} & \text{if } \overline{F}(a,b) \neq \emptyset \end{cases} \tag{1.8}$$

where, $\overline{F}(a,b) = \{j \in F / D_j(a,b) > C(a,b)\}$.

The formula 1.8, determining the value of $\sigma(a,b)$ over the interval $[0,1]$, is constructed in such a way as to fulfill certain qualitative principles, and in particular, it excludes the possibility that a big loss in one criterion might be compensated by a number of small gains on the remaining criteria [52]. See an example of the calculation on page 31.

1.1.7 The ranking algorithm in ELECTRE III

The ranking algorithm is used in the exploitation procedure of the fuzzy outranking relation; it is based on the degrees of credibility of each action in order to get a final partial pre-order, resulting from the intersection of two complete pre-orders. This algorithm needs an additional information related to the distillation, i.e., the distillation threshold function, $s(\lambda)$. This function is used to make successive cuts of the fuzzy outranking relations in order to obtain crispy outranking relations (it should be notice that the results will be influenced by the distillations thresholds and the cutoff levels chosen). For more details, consult [48, p. 415-422]. See an example of the calculation on page 39.

The two complete pre-orders are constructed in a different way. The first one is obtained in a descending manner by starting with the best action, and finishing with the assignment of the worst one; it is called *descending distillation*. The second one is obtained in a ascending manner by starting with the worst rated action, and finishing with the assignment of the best one; it is called *ascending distillation*.

To establish these pre-orders, we proceed in the following way:

- From the fuzzy outranking matrix $[\sigma(a,b)]$, a succession of crispy outranking relations is built. To do this, a set of cutoff levels, $\lambda_k \in [0,1]$, and a distillation threshold, $s(\lambda_k) = \alpha \times \lambda_k + \beta$, are defined. Then, only the arcs (a,b) of the fuzzy outranking relation for which $\sigma(a,b) > \lambda_k$ are kept, and a crispy outranking relation $S_A^{\lambda_k}$ is obtained; it can be defined as follows:

$$a\, S_A^{\lambda_k}\, b \;\Leftrightarrow\; \begin{cases} \sigma(a,b) > \lambda_k \\ \sigma(a,b) > \sigma(b,a) + s(\sigma(a,b)) \end{cases} \tag{1.9}$$

This means that the assertion "*a* outranks *b*" will be taken into account if it is more significative than the reverse assertion "*b* outranks *a*".

- From the crispy outranking matrix, the following calculation are made, for all actions:

  . **the λ_k-power of** a, $p_A^{\lambda_k}(a)$: is the number of actions that are outranked by a (it expresses for how much an action a outranks all the others).

  $$p_A^{\lambda_k}(a) = \left|\left\{b \in A \;/\; a\, S_A^{\lambda_k}\, b\right\}\right| = \text{card}\left\{b \in A \;/\; a\, S_A^{\lambda_k}\, b\right\} \tag{1.10}$$

  . **the λ_k-weakness of** a, $f_A^{\lambda_k}(a)$: is the number of actions that outrank a (expresses for how much an action a is outranked by all the others).

  $$f_A^{\lambda_k}(a) = \left|\left\{b \in A \;/\; b\, S_A^{\lambda_k}\, a\right\}\right| = \text{card}\left\{b \in A \;/\; b\, S_A^{\lambda_k}\, a\right\} \tag{1.11}$$

  . **the λ_k-qualification of** a with respect to the set A, $q_A^{\lambda_k}(a)$. This indicator expresses, in a clear way, the relative positions of the actions of the set A.

  $$q_A^{\lambda_k}(a) = p_A^{\lambda_k}(a) - f_A^{\lambda_k}(a) \tag{1.12}$$

- Let λ_1 be the first fixed cutoff level and $q_A^{\lambda_k}(a)$ be the qualification of action a. Then, select, in the set of the actions to rank, A, the best one (or the best ones in case of *ex-æquo*) obtaining thus a subset of actions from A which has the maximum qualification (descending selection, $\overline{D}_1$) or the worst action (or the worst actions) obtaining thus a subset of actions from A which has the minimum qualification (ascending selection, $\underline{D}_1$):

$$\overline{D}_1 = \left\{a \in A \;/\; q_A^{\lambda_1}(a) = \overline{q}_A = \max_{x \in A} q_A^{\lambda_1}(x)\right\} \tag{1.13}$$

$$\underline{D}_1 = \left\{a \in A \;/\; q_A^{\lambda_1}(a) = \underline{q}_A = \min_{x \in A} q_A^{\lambda_1}(x)\right\} \tag{1.14}$$

- Therefore, at the end of the k steps of the first distillation, a first subset of A which will constitute the first (or the last) class of one of the two final pre-orders has been selected. Let $\overline{C}_1 = \overline{D}_k$ denote the first class of the descending selection, and $\underline{C}_1 = \underline{D}_k$

denote the last class of the ascending selection. Let $A_1 = A \setminus \overline{C}_1$, or $A_1 = A \setminus \underline{C}_1$ denote the remaining subset of the actions from A to rank after the first distillation. In this subset, the qualification of each action is calculated again for selecting one or several actions. Reiterate until all the actions are ranked.

The ranking algorithm can be stated as follows:

1) $n = 0,\ \overline{A}_0 = A$ **or** $\underline{A}_0 = A$.

2) $\lambda_0 = \max\limits_{\substack{a,\,b\,\in\,\overline{A}_n \\ a \neq b}} \sigma(a,b)$ **or** $\lambda_0 = \max\limits_{\substack{a,\,b\,\in\,\underline{A}_n \\ a \neq b}} \sigma(a,b)$

3) $k = 0,\ D_0 = \overline{A}_n$ **or** $D_0 = \underline{A}_n$.

4) Among all the arcs of the fuzzy outranking relation which credibility is lower than $\lambda_k - s(\lambda_k)$, and choose the one that have the maximum value:

$$\lambda_{k+1} = \max_{\substack{\{\sigma(a,b) > \lambda_k - s(\lambda_k)\} \\ a,\,b\,\in\,D_k}} \sigma(a,b)$$

Notice that $\forall\, a,b \in D_k, \sigma(a,b) > \lambda_k - s(\lambda_k) \Rightarrow \lambda_{k+1} = 0$

5) Calculate the λ_k-qualifications of all the actions belonging to D_k.

6) Obtain the maximum or minimum λ_k-qualifications : $\overline{q}_{D_k}$, or $\underline{q}_{D_k}$.

7) Build the subset:

$$\overline{D}_{k+1} = \left\{ a \in D_k \ / \ q_{D_k}^{\lambda_{k+1}}(a) = \overline{q}_{D_k} \right\}$$

or

$$\underline{D}_{k+1} = \left\{ a \in D_k \ / \ q_{D_k}^{\lambda_{k+1}}(a) = \underline{q}_{D_k} \right\}$$

8) **if** $|\overline{D}_{k+1}| = 1$ **or** $|\underline{D}_{k+1}| = 1$ **or** $\lambda_{k+1} = 0$ **then**, **go to** 9

else, **do** $k = k+1,\ D_k = \overline{D}_k$ OR $D_k = \underline{D}_k$, **go to** 4

9) $\overline{C}_{n+1} = \overline{D}_{n+1}$

$\underline{C}_{n+1} = \underline{D}_{n+1}$

do $\overline{A}_{n+1} = \overline{A}_n \setminus \overline{C}_{n+1}$ **or** $\underline{A}_{n+1} = \underline{A}_n \setminus \underline{C}_{n+1}$

if $\overline{A}_{n+1} = \emptyset$ **or** $\underline{A}_{n+1} = \emptyset$ **then** $n = n+1$, **go to** 2

else, END of the distillation.

Notice that during the same distillations, when moving from step k to step $k+1$, the cutoff level λ_k is replaced by $\lambda_{k+1} < \lambda_k$ in the following transformation (let D_k be the remaining set of actions to rank):

$$\lambda_{k+1} = \max_{\substack{\{\sigma(a,b) > \lambda_k - s(\lambda_k)\} \\ a,\, b \in D_k}} \sigma(a,b) \tag{1.15}$$

where, $s(\lambda_k) = \alpha \times \lambda_k + \beta$. The user can fix one value for the distillations coefficients α and β before the calculation. However, some standard values are proposed to him/her ($\alpha = -0.15$ and $\beta = 0.30$). When using ELECTRE IV, the distillations coefficients α and β are fixed ($\alpha = 0$ and $\beta = 0.1$) in order to be coherent with the values assigned to the dominance relations (1, 0.8, 0.6, 0.4 and 0.2 for the relations S_q, S_c, S_p, S_s, S_v, respectively). See section 1.2.3 for more details.

At the end of the ascending and descending distillations, the results are two complete pre-order. In each of them, the actions are regrouped in a set of ranked equivalence classes. Each class contains at least one action. The **intersection pre-order** provides the comparisons between actions and underlines the possible incomparabilities:

- an action a will be considered better than b if in at least one of the distillations, a is better than b, and in the other distillation, a is at least as well ranked as b.
- an action a will be judged indifferent to b if the two actions belong to the same equivalence class in the two pre-orders.
- the actions a and b are incomparable if a is better ranked than b in the ascending distillation and b is better ranked than a in the descending distillation or *vice-versa*.

1.2 The ELECTRE IV method

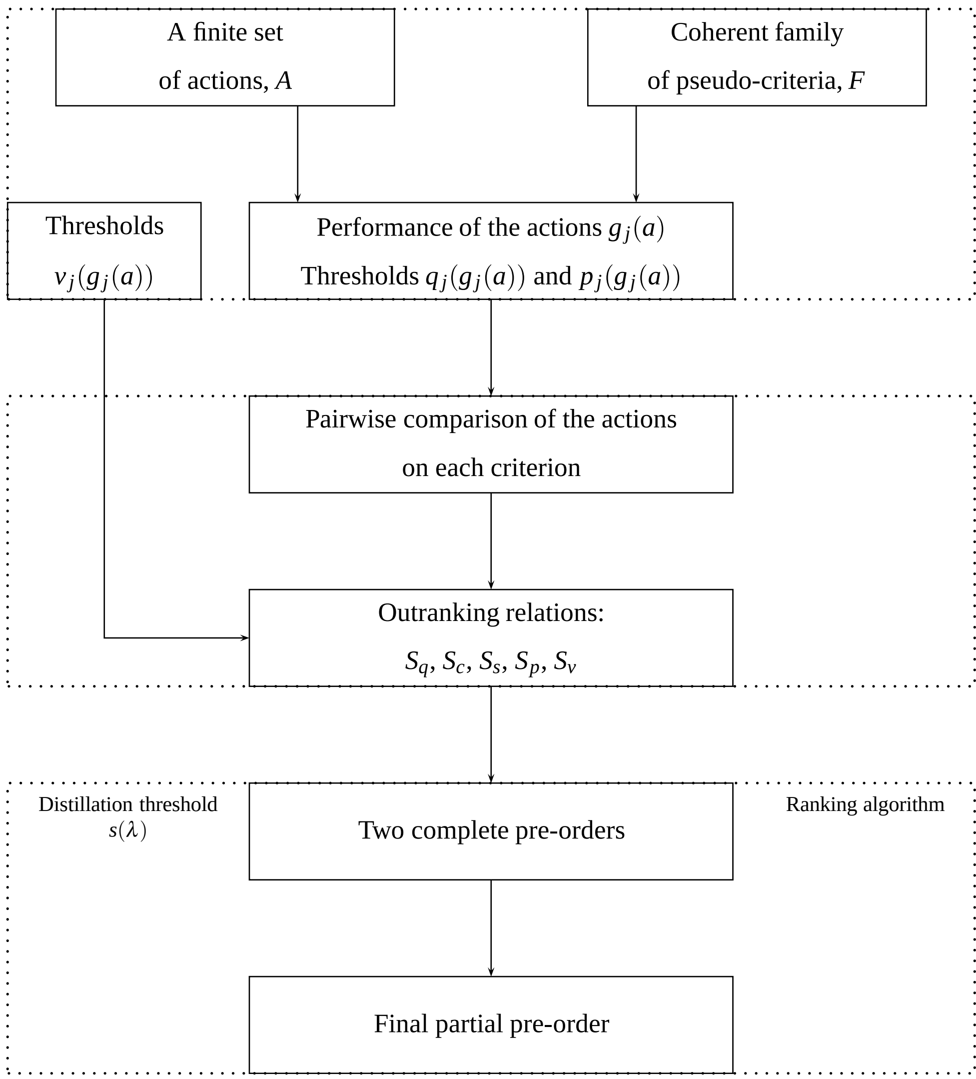


Figure 1.8: General structure of ELECTRE IV

ELECTRE IV is appropriate for the cases in which the user is not able, does not want, or does not know how to introduce information about the relative importance of the criteria. Thus, in the ELECTRE IV method no w_j is introduced. It does not mean that each criterion has exactly the same *weight* [43]. For this reason it is impossible to construct the concordance matrix.

ELECTRE IV method uses five outranking relations to build the nested outranking relations (S_q, S_c, S_p, S_s and S_v). The use of this five relations must fulfill two constraints:

- no criterion is preponderant when compared to any subset of half of the criteria.
- no criterion is negligible when compared to any subset of half of the criteria.

By pairwise comparison, if there are more than one relation, the richer one should be kept. The ELECTRE IV method exploitation procedure is the same as in ELECTRE III (Figure 1.8). For more details, see section 1.1.7 on page 15.

1.2.1 Notation

The notation used for ELECTRE III is valid for ELECTRE IV, except the one related to the weights of criteria. It is useful to introduce some specific notation:

$n_p(a,b)$: the number of criteria for which a is strictly preferred to b, $a\ P_j\ b$ (it means that are enough reasons to justify the preference of a over b).

$n_p(b,a)$: the number of criteria for which b is strictly preferred to a, $b\ P_j\ a$ (it means that are enough reasons to justify the preference of b over a).

$n_q(a,b)$: the number of criteria for which a is weakly preferred to b , $a\ Q_j\ b$ (it means that are an hesitation between strictly preference and indifference of a over b).

$n_q(b,a)$: the number of criteria for which b is weakly preferred to a, $b\ Q_j\ a$ (it means that are an hesitation between strictly preference and indifference of b over a).

$n_i(a,b)$: the number of criteria for which a is considered indifferent to b, $a\ I_j\ b$, but such that a has a better performance than b (it means that are enough reasons to justify the indifference between both actions).

$n_i(b,a)$: the number of criteria for which b is considered indifferent to a, $b\ I_j\ a$, but such that b has a better performance than a (it means that are enough reasons to justify the indifference between both actions).

$n_0(a,b)$ $= n_0(b,a)$: the number of criteria for which a and b have the same performance, $g_j(a) = g_j(b)$.

n $= n_p(a,b) + n_q(a,b) + n_i(a,b) + n_0(a,b) + n_i(b,a) + n_q(b,a) + n_p(b,a)$: the total number of criteria ($|F| = n$).

1.2.2 The five outranking relations

The conditions 1.16 - 1.21 represent the five outranking relations that are defined for ELECTRE IV method as follows [67, p. 40-41], [48, p. 270-271]:

- **Quasi-dominance (S_q)**:

$$b \; S_q \; a \Leftrightarrow \begin{cases} n_p(a,b) + n_q(a,b) = 0 \\ n_i(a,b) < n_i(b,a) + n_q(b,a) + n_p(b,a) \end{cases} \tag{1.16}$$

  The pair (b,a) verifies the relation of quasi-dominance if and only if:

  - for every criterion, b is either preferred or indifferent to a, and
  - the number of criteria for which the performance of a is better than the performance of b (a staying indifferent to b) is strictly lower than the number of criteria for which the performance of b is better than the performance of a.

- **Canonic-dominance (S_c)**:

$$b \; S_c \; a \Leftrightarrow \begin{cases} n_p(a,b) = 0 \\ n_q(a,b) \leq n_p(b,a) \\ n_q(a,b) + n_i(a,b) < n_i(b,a) + n_q(b,a) + n_p(b,a) \end{cases} \tag{1.17}$$

  The pair (b,a) verifies the relation of canonic-dominance if and only if:

  - for no criterion, a is strictly preferred to b, and
  - the number of criteria for which a is weakly preferred to b is lower than or equal to the number of criteria for which b is strictly preferred to a, and
  - the number of criteria for which the performance of a is better than the performance of b is strictly lower than the number of criteria for which the performance of b is better than the performance of a.

  $\therefore \; S_q \subset S_c$ (S_c is richer than S_q) and $S_q = S_c$ if $\{j \in F \mid a \; Q_j \; b\} = \emptyset$, i.e. $n_q(a,b) = 0$.

- **Pseudo-dominance (S_p)**:

$$b \; S_p \; a \Leftrightarrow \begin{cases} n_p(a,b) = 0 \\ n_q(a,b) \leq n_q(b,a) + n_p(b,a) \end{cases} \tag{1.18}$$

  The pair (b,a) verifies the relation of pseudo-dominance if and only if:

- for no criterion, a is strictly preferred to b, and
- the number of criteria for which a is weakly preferred to b is lower than or equal to the number of criteria for which b is strictly or weakly preferred to a.

$\therefore\ S_c \subset S_p$ (S_p is richer than S_c).

- **Sub-dominance (S_s)**:

$$b\, S_s\, a \Leftrightarrow n_p(a,b) = 0 \tag{1.19}$$

The pair (b,a) verifies the relation of sub-dominance if and only if for no criterion, a is strictly preferred to b.

$\therefore\ S_p \subset S_s$ (S_s is richer than S_p)

- **Veto-dominance (S_v)**:

$$b\, S_v\, a \Leftrightarrow n_p(a,b) = 0 \tag{1.20}$$

or

$$b\, S_v\, a \Leftrightarrow \begin{cases} n_p(a,b) = 1 \\ n_p(b,a) \geq \frac{n}{2} \\ g_j(b) + v_j(g_j(b)) \geq g_j(a), \forall\, j\ \in\ F \end{cases} \tag{1.21}$$

The pair (b,a) verifies the relation of veto-dominance if and only if:

- either for no criterion, a is strictly preferred to b, or
- a is strictly preferred to b for only one criterion but this criterion does not veto the outranking of a over b and furthermore, b is strictly preferred to a for at least half of the criteria.

  $\therefore\ S_s \subset S_v$ (S_v is richer than S_s)

1.2.3 The fuzzy outranking relations in ELECTRE IV

For each pair of actions (a,b), the credibility degrees are in the range $[0,1]$ that indicate in what measure it can be affirmed that "a outranks b". For the calculation of the credibility index, $\sigma(a,b)$ the following indicators must be calculated for the assignment of the relations to the pair (a,b): $n_p(a,b)$, $n_q(a,b)$, $n_i(a,b)$, $n_0(a,b)$, $n_i(b,a)$, $n_q(b,a)$, and $n_p(b,a)$. If there are more than one relation, the richer one should be kept. See an example of the calculation on page 32.

For each dominance relation there is a value for the credibility degrees, in order to build the credibility matrix. These values (expression 1.22), together with the coefficients of the distillation threshold function, were built in order to be coherent with the distillation mechanism in the ranking algorithm and the successive cuts on the ELECTRE IV relations.

$$\sigma(a,b) = \begin{cases} 1 & \text{if } a\ S_q\ b \text{ (quasi-dominance)} \\ 0.8 & \text{if } a\ S_c\ b \text{ (canonic-dominance)} \\ 0.6 & \text{if } a\ S_p\ b \text{ (pseudo-dominance)} \\ 0.4 & \text{if } a\ S_s\ b \text{ (sub-dominance)} \\ 0.2 & \text{if } a\ S_v\ b \text{ (veto-dominance)} \\ 0 & \text{if no relation among } \{S_q, S_c, S_p, S_s, S_v\} \text{ for } (a,b) \end{cases} \tag{1.22}$$

The distillation threshold function, $s(\lambda) = \alpha \times \lambda + \beta$, used in ELECTRE IV, is constant, where $\alpha = 0$ and $\beta = 0.1$. Thus, $s(\lambda) = 0.1$. This value allows the transformation of a nested relation into a fuzzy one. As consequences:

- in the first step of the ranking, only the strongest dominance among those that have been verified will be taken into account.
- in the second step of the ranking, the two strongest dominance will be taken into account, etc.

1.3 An illustrative example

This illustrative example has been used in [67, 68] to present the ELECTRE III-IV software in the French version. The objective is to make the ranking of 10 French cars that were evaluated on 7 criteria (Tables 1.1, 1.2, 1.3, and 1.4).

In Table 1.2, the line *Mode of definition* indicates the mode of calculation of the thresholds (→, **direct**, considers the worst of the two actions; ←, **inverse**, considers the best of the two actions). The lines concerning the *indifference*, *preference*, and *veto* present the coefficients

α and β of the affine function for the calculation of the thresholds: $q_j(g_j(a)) = \alpha_j \times g_j(a) + \beta_j$, $p_j(g_j(a)) = \alpha_j \times g_j(a) + \beta_j$, and $v_j(g_j(a)) = \alpha_j \times g_j(a) + \beta_j$. In the next paragraphs, some ELECTRE III and ELECTRE IV calculations are placed to this illustrative example.

Table 1.1: List of criteria and codes

Criterion name	Code
Price	g_1: Prix
Maximum power per Km/h	g_2: Vmax
Consumption in 120 Km/h per litre	g_3: C120
Volume of the case per dm3	g_4: Coff
0.100 Km/h in seconds	g_5: Acce
Distance of braking by 130 km/h	g_6: Frei
Sound level in db	g_7: Brui

Table 1.2: Definition of the pseudo-criteria

	g_1 Prix	g_2 Vmax	g_3 C120	g_4 Coff	g_5 Acce	g_6 Frei	g_7 Brui
Direction of preference	Decreasing $\downarrow$	Increasing $\uparrow$	Decreasing $\downarrow$	Increasing $\uparrow$	Decreasing $\downarrow$	Decreasing $\downarrow$	Decreasing $\downarrow$
Weight, w_j	0.3	0.1	0.3	0.2	0.1	0.2	0.1
Mode of definition	Inverse $\leftarrow$	Direct $\rightarrow$	Direct $\rightarrow$	Direct $\rightarrow$	Direct $\rightarrow$	Direct $\rightarrow$	Direct $\rightarrow$
Indifference (α / β)	0.08 / -2000	0.02 / 0	0 / 1	0 / 100	0.1 / -0.5	0 / 0	0 / 3
Preference (α / β)	0.13 / -3000	0.05 / 0	0 / 2	0 / 200	0.2 / -1	0 / 5	0 / 5
Veto (α / β)	0.9 / 50000	-	0 / 4	-	0.5 / 3	0 / 15	0 / 15

Table 1.3: List of actions and codes

Alternative name	Code
Citroen BX 16 TZS	a_1: CBX16
Peugeot 205 GTI 1.9	a_2: P205G
Peugeot 405 MI16	a_3: P405M
Peugeot 605 SV 24	a_4: P605S
Renault 4 GTL CLAN	a_5: R4GTL
Renault CLIO 16S	a_6: RCLIO
Renault 21 TSE	a_7: R21TS
Renault 21 2L.TURBO	a_8: R21TU
Renault 25 BACCARA V6	a_9: R25BA
Renault ALPINE A610 TURBO	a_{10}: ALPIN

Table 1.4: Table of performances

	g_1 Prix	g_2 Vmax	g_3 C120	g_4 Coff	g_5 Acce	g_6 Frei	g_7 Brui
a_1: CBX16	103000	171.3	7.65	352	11.6	88.0	69.7
a_2: P205G	101300	205.3	7.90	203	8.4	78.3	73.4
a_3: P405M	156400	221.7	7.90	391	8.4	81.5	69.0
a_4: P605S	267400	230.7	10.50	419	8.6	64.7	65.6
a_5: R4GTL	49900	122.6	8.30	120	23.7	74.1	76.4
a_6: RCLIO	103600	205.1	8.20	265	8.1	81.7	73.6
a_7: R21TS	103000	178.0	7.20	419	11.4	77.6	66.2
a_8: R21TU	170100	226.0	9.10	419	8.1	74.7	71.7
a_9: R25BA	279700	233.8	10.90	359	7.8	75.5	70.9
a_{10}: ALPIN	405000	265.0	10.30	265	6.0	74.7	72.0

Concordance indices

According to the illustrative example, the calculations of the concordances indices can be made as follows [67, p. 21-28]:

Example 1: Calculations of $\boldsymbol{C_4(a_2,a_1)}$, $\boldsymbol{C_4(a_2,a_6)}$, and $\boldsymbol{C_4(a_2,a_7)}$

Table 1.5: Comparison of actions on criterion g_4

	g_4: Coff ($\uparrow \rightarrow$)
a_1: CBX16	352
a_2: P205G	**203**
a_6: RCLIO	265
a_7: R21TS	419

On criterion g_4, the preferences are in increasing direction with the performances, and the thresholds are direct, and constant ($\alpha = 0$):

- $q_4(g_4(a_2)) = 100$
- $p_4(g_4(a_2)) = 200$
- $g_4(a_2) + q_4(g_4(a_2)) = 203 + 100 = 303$
- $g_4(a_2) + p_4(g_4(a_2)) = 203 + 200 = 403$

- $g_4(a_6) < g_4(a_2) + q_4(g_4(a_2)) \Rightarrow \boldsymbol{C_4(a_2, a_6) = 1}$

- $g_4(a_7) > g_4(a_2) + p_4(g_4(a_2)) \Rightarrow \boldsymbol{C_4(a_2, a_7) = 0}$

- $g_4(a_2) + q_4(g_4(a_2)) < g_4(a_1) < g_4(a_2) + p_4(g_4(a_2))$ means that the calculation of $C_4(a_2, a_1)$ is made by linear interpolation:

$$\boldsymbol{C_4(a_2, a_1) =} \frac{p_4(g_4(a_2)) - [g_4(a_1) - g_4(a_2)]}{p_4(g_4(a_2)) - q_4(g_4(a_2))} = \frac{200 - (352 - 203)}{200 - 100} \boldsymbol{= 0.51}$$

Example 2: Calculations of $\boldsymbol{C_2(a_3, a_4)}$, $\boldsymbol{C_2(a_3, a_5)}$, and $\boldsymbol{C_2(a_3, a_9)}$

Table 1.6: Comparison of actions on criterion g_2

	g_2: Vmax (↑ →)
a_3: P405M	**221.7**
a_4: P605S	230.7
a_5: R4GTL	<u>122.6</u>
a_9: R25BA	233.8

On criterion g_2, the preferences are in increasing direction with the performances, and the thresholds are direct. The action a_3 is better than action a_5 on criterion g_2. Thus, $\boldsymbol{C_2(a_3, a_5) = 1}$. It remains the comparison of the action a_3 to a_4, and a_9:

- $q_2(g_2(a_3)) = 0.02 \times 221.7 + 0 = 4.434$

- $p_2(g_2(a_3)) = 0.05 \times 221.7 + 0 = 11.085$

- $g_2(a_3) + q_2(g_2(a_3)) = 221.7 + 4.434 = 226.134$

- $g_2(a_3) + p_2(g_2(a_3)) = 221.7 + 11.085 = 232.785$

- $g_2(a_9) > g_2(a_3) + p_2(g_2(a_3)) \Rightarrow \boldsymbol{C_2(a_3, a_9) = 0}$

- $g_2(a_3) + q_2(g_2(a_3)) < g_2(a_4) < g_2(a_3) + p_2(g_2(a_3))$ means that the calculation of $C_2(a_3, a_4)$ is made by linear interpolation:

$$\boldsymbol{C_4(a_2, a_1) =} \frac{232.785 - 230.7}{11.085 - 4.434} \boldsymbol{= 0.31}$$

Table 1.7: Comparison of actions on criterion g_2

	g_6: Frei (↓ →)
a_1: CBX16	<u>88</u>
a_2: P205G	78.3
a_6: RCLIO	**81.7**
a_7: R21TS	77.6
a_8: R21TU	74.7

Example 3: Calculations of $\boldsymbol{C_6(a_6,a_1)}$, $\boldsymbol{C_6(a_6,a_2)}$, $\boldsymbol{C_6(a_6,a_7)}$, and $\boldsymbol{C_6(a_6,a_8)}$

On criterion g_6, the preferences are in decreasing direction with the performances, and the thresholds are direct, and constant ($\alpha = 0$). The action a_6 has a better performance than the action a_1. Thus, $\boldsymbol{C_6(a_6,a_1) = 1}$. For the remaining actions:

- $q_6(g_6(a_6)) = 0$
- $p_6(g_6(a_6)) = 5$
- $g_6(a_6) - q_6(g_6(a_6)) = 81.7 - 0 = 81.7$
- $g_6(a_6) - p_6(g_6(a_6)) = 81.7 - 5 = 76.7$
- $g_6(a_6) - p_6(g_6(a_6)) > g_6(a_8) \Rightarrow \boldsymbol{C_2(a_6,a_8) = 0}$
- $g_6(a_6) - p_6(g_6(a_6)) < g_6(a_7) < g_2(a_6) - q_6(g_6(a_6))$ means that the calculation of $C_6(a_6,a_7)$ is made by linear interpolation:

  $$\boldsymbol{C_6(a_6,a_7)} = \frac{77.6 - 76.7}{5 - 0} = \boldsymbol{0.18}$$

- $g_6(a_6) - p_6(g_6(a_6)) < g_6(a_2) < g_2(a_6) - q_6(g_6(a_6))$ means that the calculation of $C_6(a_6,a_2)$ is made by linear interpolation:

  $$\boldsymbol{C_6(a_6,a_2)} = \frac{78.3 - 76.7}{5 - 0} = \boldsymbol{0.32}$$

Example 4: Calculations of $\boldsymbol{C_1(a_8,a_3)}$

On criterion g_1, the preferences are in decreasing direction with the performances, and the thresholds are inverse. The ELECTRE III-IV software transforms the inverse thresholds into direct ones, based on the inverse coefficients given by the decision-maker. If α', and β' are the coefficients of the inverse threshold; α, and β the coefficients of the direct thresholds:

. $\alpha_q = \dfrac{\alpha_q'}{1+\alpha_q'} = \dfrac{0.08}{1+0.08} = 0.074$

. $\beta_q = \dfrac{\beta_q'}{1+\alpha_q'} = \dfrac{-2000}{1+0.08} = -1851.85$

. $\alpha_p = \dfrac{\alpha_p'}{1+\alpha_p'} = \dfrac{0.13}{1+0.13} = 0.115$

. $\beta_p = \dfrac{\beta_p'}{1+\alpha_p'} = \dfrac{-3000}{1+0.13} = -2654.867$

. $q_1(g_1(a_8)) = \alpha_q \times g_1(a_8) + \beta_q = 0.074 \times 170100 + (-1851.85) = 10735.55$

. $g_1(a_8) - q_1(g_1(a_8)) = 170100 - 10735.55 = 159364.45$

. $p_1(g_1(a_8)) = \alpha_p \times g_1(a_8) + \beta_p = 0.115 \times 170100 + (-2654.867) = 16906.63$

. $g_1(a_8) - p_1(g_1(a_8)) = 170100 - 16906.63 = 153193.37$

. $g_1(a_8) - p_1(g_1(a_8)) < g_1(a_3) < g_1(a_8) - q_1(g_1(a_8))$ means that the calculation of $C_1(a_8, a_3)$ is made by linear interpolation:

$$\boldsymbol{C_1(a_8, a_3) =} \frac{156400 - 153193.37}{16906.63 - 10735.55} \boldsymbol{= 0.52}$$

Table 1.8: Comparison of actions on criterion g_1

	g_1: Prix ($\downarrow \leftarrow$)
a_3: P205M	156400
a_8: R21TU	**170100**

In any way, in the calculation process it is possible to obtain the concordance indices $C_j(a,b)$ on criterion g_j for all pair of actions (a,b), and finally displaying the concordance matrices for each criterion. For instance, on criterion g_1, the concordance matrix is defined in Table 1.9.

Comprehensive concordance indices

According to the illustrative example, the values of $\boldsymbol{C(a_9, a_8)}$, $\boldsymbol{C(a_1, a_4)}$, and $\boldsymbol{C(a_3, a_8)}$ are calculated as follows [67, p. 29]:

$$C(a_9,a_8) = \frac{(0.3\times 0)+(0.1\times 1)+(0.3\times 0.2)+(0.2\times 1)+(0.1\times 1)+(0.2\times 0.84)+(0.1\times 1)}{0.3+0.1+0.3+0.2+0.1+0.2+0.1} = 0.56$$

$$C(a_1,a_4) = \frac{(0.3\times 1)+(0.1\times 0)+(0.3\times 1)+(0.2\times 1)+(0.1\times 0)+(0.2\times 0)+(0.1\times 0.45)}{0.3+0.1+0.3+0.2+0.1+0.2+0.1} = 0.65$$

$$C(a_3,a_8) = \frac{(0.3\times 1)+(0.1\times 1)+(0.3\times 1)+(0.2\times 1)+(0.1\times 1)+(0.2\times 0)+(0.1\times 1)}{0.3+0.1+0.3+0.2+0.1+0.2+0.1} = 0.846$$

For all pairs of actions representing the illustrative example, the comprehensive concordance matrix is obtained (Table 1.10). This matrix can also be found in the next chapter (Figure 2.32 on page 90).

Table 1.9: Concordance matrix on criterion g_1

$\sigma(.,.)$	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
a_1	1	1	1	1	0	1	1	1	1	1
a_2	1	1	1	1	0	1	1	1	1	1
a_3	0	0	1	1	0	0	0	1	1	1
a_4	0	0	0	1	0	0	0	0	1	1
a_5	1	1	1	1	1	1	1	1	1	1
a_6	1	1	1	1	0	1	1	1	1	1
a_7	1	1	1	1	0	1	1	1	1	1
a_8	0	0	0.52	1	0	0	0	1	1	1
a_9	0	0	0	1	0	0	0	0	1	1
a_{10}	0	0	0	0	0	0	0	0	0	1

Table 1.10: Comprehensive concordance matrix

$C(.,.)$	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
a_1	1	0.69	0.69	0.65	0.62	0.69	0.78	0.69	0.69	0.69
a_2	0.9	1	0.73	0.54	0.64	1	0.75	0.66	0.69	0.74
a_3	0.77	0.67	1	0.78	0.62	0.77	0.65	0.85	0.71	0.69
a_4	0.54	0.54	0.54	1	0.54	0.51	0.54	0.65	0.92	0.85
a_5	0.62	0.85	0.62	0.46	1	0.78	0.59	0.63	0.62	0.72
a_6	0.97	0.9	0.82	0.61	0.62	1	0.71	0.69	0.77	0.69
a_7	1	0.85	0.85	0.69	0.66	0.85	1	0.76	0.78	0.76
a_8	0.67	0.72	0.84	0.77	0.75	0.77	0.48	1	0.96	0.85
a_9	0.54	0.54	0.54	0.77	0.5	0.54	0.47	0.56	1	0.82
a_{10}	0.54	0.54	0.5	0.46	0.52	0.54	0.38	0.64	0.77	1

Discordance indices

In the illustrative example, the veto thresholds have been defined on criteria g_1, g_3, g_5, g_6, and g_7. Only these criteria can give a discordance index not null [67, p. 32-35].

Example 1: Calculations of $\boldsymbol{D_6(a_1,a_4)}$, and $\boldsymbol{D_6(a_1,a_7)}$

Table 1.11: Comparison of actions on criterion g_6

	g_6: Prix ($\downarrow \rightarrow$)
a_1: CBX16	**88**
a_4: P605S	64.7
a_7: R21TS	77.6

On criterion g_6, the preferences are in decreasing direction with the performances, and the thresholds are direct, and constant ($\alpha = 0$):

- $p_6(g_6(a_1)) = 5$
- $v_6(g_6(a_1)) = 15$
- $g_6(a_1) - g_6(a_4) \geq v_6(g_6(a_1)) \Rightarrow \boldsymbol{D_6(a_1,a_4) = 1}$
- $g_6(a_1) - v_6(g_6(a_1)) < g_6(a_7) < g_6(a_1) - p_6(g_6(a_1))$ means that the calculation of $D_6(a_1,a_7)$ is made by linear interpolation:

$$\boldsymbol{D_6(a_1,a_7)} = \frac{g_6(a_1) - p_6(g_6(a_1)) - g_6(a_7)}{v_6(g_6(a_1)) - p_6(g_6(a_1))} = \frac{83 - 77.6}{15 - 5} = \boldsymbol{0.54}$$

Example 2: Calculations of $\boldsymbol{D_1(a_9,a_1)}$, $\boldsymbol{D_1(a_9,a_4)}$, and $\boldsymbol{D_1(a_9,a_8)}$

Table 1.12: Comparison of actions on criterion g_1

	g_1: Prix ($\downarrow \leftarrow$)
a_1: CBX16	103000
a_4: P605S	267400
a_8: R21TU	170100
a_9: R25BA	**279700**

On criterion g_1, the preferences are in decreasing direction with the performances, and the thresholds are inverse. The ELECTRE III-IV software transforms the inverse thresholds into direct ones, based on the inverse coefficients given by the decision-maker. If α', and β' are the coefficients of the inverse threshold; α, and β the coefficients of the direct thresholds:

- $\alpha_p = \frac{\alpha_p'}{1+\alpha_p'} = \frac{0.13}{1+0.13} = 0.115$
- $\beta_p = \frac{\beta_p'}{1+\alpha_p'} = \frac{-3000}{1+0.13} = -2654.87$
- $\alpha_v = \frac{\alpha_v'}{1+\alpha_v'} = \frac{0.9}{1+0.9} = 0.474$
- $\beta_v = \frac{\beta_v'}{1+\alpha_v'} = \frac{50000}{1+0.9} = 26315.79$
- $p_1(g_1(a_9)) = \alpha_p \times g_1(a_9) + \beta_p = 0.115 \times 279700 + (-2654.867) = 29510.63$
- $g_1(a_9) - p_1(g_1(a_9)) = 279700 - 29510.63 = 250189.37$
- $v_1(g_1(a_9)) = \alpha_v \times g_1(a_9) + \beta_v = 0.474 \times 279700 + 26315.79 = 158893.59$
- $g_1(a_9) - v_1(g_1(a_9)) = 279700 - 158893.59 = 120806.41$
- $g_1(a_9) - g_1(a_4) < p_1(g_1(a_9)) \Rightarrow \boldsymbol{D_1(a_9,a_4) = 0}$
- $g_1(a_9) - g_1(a_1) > v_1(g_1(a_9)) \Rightarrow \boldsymbol{D_1(a_9,a_1) = 1}$
- $g_1(a_9) - v_1(g_1(a_9)) < g_1(a_8) < g_1(a_9) - p_1(g_1(a_9))$ means that the calculation of $D_1(a_9,a_8)$ is made by linear interpolation:

$$\boldsymbol{D_1(a_9,a_8)} = \frac{g_1(a_9) - p_1(g_1(a_9)) - g_1(a_8)}{v_1(g_61(a_9)) - p_1(g_1(a_9))} = \frac{250189.37 - 170100}{158893.59 - 29510.63} = \mathbf{0.62}$$

In any way, in the calculation process it is possible to obtain the discordance matrix $[D_j(a,b)]$ on criterion g_j for all pair of actions (a,b).

Fuzzy outranking relation in ELECTRE III

According to the comprehensive concordance matrix, and the partial discordance matrices, the values of $\boldsymbol{\sigma(a_1,a_7)}$, $\boldsymbol{\sigma(a_1,a_4)}$, and $\boldsymbol{\sigma(a_9,a_8)}$ are calculated as follows [67, p. 36]:

- $\overline{F}(a_1,a_7) = \emptyset$, with $\forall j,\ D_j(a_1,a_7) < C(a_1,a_7) \Rightarrow \sigma(a_1,a_7) = C(a_1,a_7) = 0.78$

. $D_6(a_1,a_4) = 1 \Rightarrow \sigma(a_1,a_4) = 0$

. $C(a_9,a_8) = 0.56$, $D_1(a_9,a_8) = 0.62$, and for all $j \neq 1$, $D_j(a_9,a_8) = 0 \Rightarrow \overline{F}(a_9,a_8) = \{g_1\} \Rightarrow \sigma(a_9,a_8) = 0.56 \times \dfrac{1-0.62}{1-0.56} = 0.48$

For all pairs of actions representing the illustrative example, the credibility matrix, or fuzzy outranking matrix is obtained (Table 1.13). This matrix can also be found in the next chapter (Figure 2.33 on page 91).

Table 1.13: Credibility matrix in ELECTRE III

$\sigma(.,.)$	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
a_1	1	0.692	0.692	0	0.153	0.692	0.778	0.382	0.562	0.382
a_2	0.898	1	0.734	0.163	0.623	1	0.748	0.659	0.692	0.735
a_3	0.769	0.671	1	0	0	0.769	0.649	0.846	0.71	0.692
a_4	0	0	0.392	1	0	0	0	0.647	0.923	0.846
a_5	0.396	0	0	0	1	0	0.333	0	0	0
a_6	0.965	0.895	0.815	0	0.525	1	0.714	0.686	0.769	0.692
a_7	1	0.846	0.846	0.472	0.654	0.846	1	0.757	0.781	0.757
a_8	0.665	0.723	0.843	0.767	0	0.769	0.416	1	0.963	0.846
a_9	0	0	0.32	0.769	0	0	0	0.484	1	0.822
a_{10}	0	0	0	0.355	0	0	0	0	0.769	1

Fuzzy outranking relation in ELECTRE IV

The next three example are used for the calculation of the fuzzy outranking relation of the ELECTRE IV method according to the illustrative example presented in the beginning of this section. Thus, for each pair of actions (a,b), and for each criterion g_j, starting with the determination of which the five relations that is verified among: $a\ P_j\ b$, $a\ Q_j\ b$, $a\ I_j\ b$, $b\ Q_j\ a$, and $b\ P_j\ a$. The calculation can be made using the inverse, or the direct thresholds, but the ELECTRE III-IV software use only direct thresholds, by transformation of the inverse ones. To simplify, on these examples, the inverse thresholds are obtained according to the best action [67, p. 42-48].

Example 1: Calculations of $\boldsymbol{\sigma(a_2,a_8)}$, and $\boldsymbol{\sigma(a_8,a_2)}$

- On criterion g_1, the preferences are in decreasing direction with the performances, and the thresholds are inverse:

$$q_1(g_1(a_2)) = 0.08 \times 101300 + (-2000) = 6104$$

$$p_1(g_1(a_2)) = 0.13 \times 101300 + (-3000) = 10169$$

$$g_1(a_8) - g_1(a_2) = 170100 - 101300 = 68800$$

$$\therefore\ g_1(a_8) - g_1(a_2) > p_1(g_1(a_2)) \Longrightarrow \boldsymbol{a_2\ P_1\ a_8 \Rightarrow n_p(a_2, a_8) := 1}$$

Table 1.14: Comparison of actions a_2 and a_8

	g_1: Prix $\downarrow \leftarrow$	g_2: Vmax $\uparrow \rightarrow$	g_3: C120 $\downarrow \rightarrow$	g_4: Coff $\uparrow \rightarrow$	g_5: Acce $\downarrow \rightarrow$	g_6: Frei $\downarrow \rightarrow$	g_7: Brui $\downarrow \rightarrow$
a_2: P205G	**101300**	**205.3**	7.9	**203**	**8.4**	**78.3**	**73.4**
a_8: R21TU	170100	226	**9.1**	419	8.1	74.7	71.7

- On criterion g_2, the preferences are in increasing direction with the performances, and the thresholds are direct:

$$q_2(g_2(a_2)) = 0.02 \times 205.3 + 0 = 4.106$$

$$p_2(g_2(a_2)) = 0.05 \times 205.3 + 0 = 10.265$$

$$g_2(a_8) - g_2(a_2) = 226 - 205.3 = 20.7$$

$$\therefore\ g_2(a_8) - g_2(a_2) > p_2(g_2(a_2)) \Longrightarrow \boldsymbol{a_8\ P_2\ a_2} \Rightarrow n_p(a_8, a_2) := 1$$

- On criterion g_3, the preferences are in decreasing direction with the performances, and the thresholds are direct, and constant ($\alpha = 0$):

$$q_3(g_3(a_8)) = 1$$

$$p_3(g_3(a_8)) = 2$$

$$g_3(a_8) - g_3(a_2) = 9.1 - 7.9 = 1.2$$

$$\therefore\ q_3(g_3(a_8)) < g_3(a_8) - g_3(a_2) < p_3(g_3(a_8)) \Longrightarrow \boldsymbol{a_2\ Q_3\ a_8 \Rightarrow n_q(a_2, a_8) := 1}$$

- On criterion g_4, the preferences are in increasing direction with the performances, and the thresholds are direct, and constant ($\alpha = 0$):

$$q_4(g_4(a_2)) = 100$$

$$p_4(g_4(a_2)) = 200$$

$$g_4(a_8) - g_4(a_2) = 419 - 203 = 216$$

$$\therefore\ g_4(a_8) - g_4(a_2) > p_4(g_4(a_2)) \Longrightarrow \boldsymbol{a_8\ P_4\ a_2 \Rightarrow n_p(a_8, a_2) := 1 + 1 = 2}$$

- On criterion g_5, the preferences are in decreasing direction with the performances, and the thresholds are direct:

$$q_5(g_5(a_2)) = 0.1 \times 8.4 + (-0.5) = 0.34$$

$$p_5(g_5(a_2)) = 0.2 \times 8.4 + (-1) = 0.68$$

$$g_5(a_2) - g_5(a_8) = 8.4 - 8.1 = 0.3$$

$$\therefore\ g_5(a_2) - g_5(a_8) < q_5(g_5(a_2)) \Longrightarrow \boldsymbol{a_2\ I_5\ a_8} \Rightarrow n_i(a_8,a_2) := 1$$

- On criterion g_6, the preferences are in decreasing direction with the performances, and the thresholds are direct, and constant ($\alpha = 0$):

$$q_6(g_6(a_2)) = 0$$

$$p_6(g_6(a_2)) = 5$$

$$g_6(a_2) - g_6(a_8) = 78.3 - 74.7 = 3.6$$

$$\therefore\ q_6(g_6(a_2)) < g_6(a_2) - g_6(a_8) < q_6(g_6(a_2)) \Longrightarrow \boldsymbol{a_8\ Q_6\ a_2} \Rightarrow \boldsymbol{n_q(a_8,a_2) := 1}$$

- On criterion g_7, the preferences are in decreasing direction with the performances, and the thresholds are direct, and constant ($\alpha = 0$):

$$q_7(g_7(a_2)) = 3$$

$$p_7(g_7(a_2)) = 5$$

$$g_7(a_2) - g_7(a_8) = 73.4 - 71.7 = 1.7$$

$$\therefore\ g_7(a_2) - g_7(a_8) < q_7(g_7(a_2)) \Longrightarrow \boldsymbol{a_2\ I_7\ a_8} \Rightarrow \boldsymbol{n_i(a_8,a_2) := 1+1 = 2}$$

Obtaining consequently the following indicators: $n_p(a_2,a_8) = 1$, $n_q(a_2,a_8) = 1$, $n_i(a_2,a_8) = 0$, $n_i(a_8,a_2) = 2$, $n_q(a_8,a_2) = 1$, and $n_p(a_8,a_2) = 2$.

Then, no dominance relation (S_q, S_c, S_p, S_s, S_v) has been found from a_2 to a_8. Thus, $\sigma(a_2,a_8) = 0$, and $\sigma(a_8,a_2) = 0$.

Example 2: Calculations of $\boldsymbol{\sigma(a_1,a_6)}$, and $\boldsymbol{\sigma(a_6,a_1)}$

- On criterion g_1, the preferences are in decreasing direction with the performances, and the thresholds are inverse:

$$q_1(g_1(a_1)) = 0.08 \times 103000 + (-2000) = 6240$$

$$p_1(g_1(a_1)) = 0.13 \times 103000 + (-3000) = 10390$$

$$g_1(a_6) - g_1(a1) = 103600 - 103000 = 600$$

$$\therefore\ g_1(a_6) - g_1(a_1) < q_1(g_1(a_1)) \Longrightarrow \boldsymbol{a_1\ I_1\ a_6} \Rightarrow n_i(a_1, a_6) := 1$$

Table 1.15: Comparison of actions a_1 and a_6

	g_1: Prix $\downarrow\leftarrow$	g_2: Vmax $\uparrow\rightarrow$	g_3: C120 $\downarrow\rightarrow$	g_4: Coff $\uparrow\rightarrow$	g_5: Acce $\downarrow\rightarrow$	g_6: Frei $\downarrow\rightarrow$	g_7: Brui $\downarrow\rightarrow$
a_1: CBX16	**103000**	**171.3**	7.65	352	**11.6**	**88**	69.7
a_6: RCLIO	103600	205.1	**8.2**	**265**	8.1	81.7	**73.6**

- On criterion g_2, the preferences are in increasing direction with the performances, and the thresholds are direct:

$$q_2(g_2(a_1)) = 0.02 \times 171.3 + 0 = 3.426$$

$$p_2(g_2(a_1)) = 0.05 \times 171.3 + 0 = 8.565$$

$$g_2(a_6) - g_2(a_1) = 205.1 - 171.3 = 33.8$$

$$\therefore\ g_2(a_6) - g_2(a_1) > p_2(g_2(a_1)) \Longrightarrow \boldsymbol{a_6\ P_2\ a_1} \Rightarrow n_p(a_6, a_1) := 1$$

- On criterion g_3, the preferences are in decreasing direction with the performances, and the thresholds are direct, and constant ($\alpha = 0$):

$$q_3(g_3(a_6)) = 1$$

$$p_3(g_3(a_6)) = 2$$

$$g_3(a_6) - g_3(a_1) = 8.2 - 7.65 = 0.55$$

$$\therefore\ g_3(a_6) - g_3(a_1) < q_3(g_3(a_6)) \Longrightarrow \boldsymbol{a_1\ I_3\ a_6} \Rightarrow n_i(a_1, a_6) := 1 + 1 = 2$$

- On criterion g_4, the preferences are in increasing direction with the performances, and the thresholds are direct, and constant ($\alpha = 0$):

$$q_4(g_4(a_6)) = 100$$

$$p_4(g_4(a_6)) = 200$$

$$g_4(a_1) - g_4(a_6) = 352 - 265 = 87$$

$$\therefore\ g_4(a_1) - g_4(a_6) < q_4(g_4(a_6)) \Longrightarrow \boldsymbol{a_1\ I_4\ a_6} \Rightarrow \boldsymbol{n_i(a_1, a_6) := 2 + 1 = 3}$$

- On criterion g_5, the preferences are in decreasing direction with the performances, and the thresholds are direct:

$$q_5(g_5(a_1)) = 0.1 \times 11.6 + (-0.5) = 0.66$$

$$p_5(g_5(a_1)) = 0.2 \times 11.6 + (-1) = 1.32$$

$$g_5(a_1) - g_5(a_6) = 11.6 - 8.1 = 3.5$$

$$\therefore\ g_5(a_1) - g_5(a_6) > p_5(g_5(a_1)) \Longrightarrow \boldsymbol{a_6\ P_5\ a_1} \Rightarrow n_p(a_6, a_1) := 1 + 1 = 2$$

- On criterion g_6, the preferences are in decreasing direction with the performances, and the thresholds are direct, and constant ($\alpha = 0$):

$$q_6(g_6(a_1)) = 0$$

$$p_6(g_6(a_1)) = 5$$

$$g_6(a_1) - g_6(a_6) = 88 - 81.7 = 6.3$$

$$\therefore\ g_6(a_1) - g_6(a_6) > p_6(g_6(a_1)) \Longrightarrow \boldsymbol{a_6\ P_6\ a_1} \Rightarrow \boldsymbol{n_p(a_6, a_1) := 2 + 1 = 3}$$

- On criterion g_7, the preferences are in decreasing direction with the performances, and the thresholds are direct, and constant ($\alpha = 0$):

$$q_7(g_7(a_6)) = 3$$

$$p_7(g_7(a_6)) = 5$$

$$g_7(a_6) - g_7(a_1) = 73.6 - 69.7 = 3.9$$

$$\therefore\ q_7(g_7(a_6)) < g_7(a_6) - g_7(a_1) < p_7(g_7(a_6)) \Longrightarrow \boldsymbol{a_1\ Q_7\ a_6} \Rightarrow \boldsymbol{n_q(a_1, a_6) := 1}$$

Obtaining consequently the following indicators: $n_p(a_1, a_6) = 0$, $n_q(a_1, a_6) = 1$, $n_i(a_1, a_6) = 3$, $n_i(a_6, a_1) = 0$, $n_q(a_6, a_1) = 0$, and $n_p(a_6, a_1) = 3$.

Since $n_p(a_1, a_6) = 0$ and $n_q(a_1, a_6) \leq n_q(a_6, a_1) + n_p(a_6, a_1)$, then $\boldsymbol{a_6\ S_p\ a_1}$. Since $n_q(a_1, a_6) + n_i(a_1, a_6) > n_i(a_6, a_1) + n_q(a_6, a_1) + n_p(a_6, a_1)$ then, $\boldsymbol{\sim (a_6\ S_c\ a_1)}$. Since $n_p(a_6, a_1) > 1$ then, no dominance relation has been found from a_1 to a_6. Thus, $\sigma(a_6, a_1) = 0.6$, and $\sigma(a_1, a_6) = 0$.

Example 3: Calculations of $\boldsymbol{\sigma(a_2, a_6)}$, and $\boldsymbol{\sigma(a_6, a_2)}$

- On criterion g_1, the preferences are in decreasing direction with the performances, and the thresholds are inverse:

$$q_1(g_1(a_2)) = 0.08 \times 101300 + (-2000) = 6104$$

$$p_1(g_1(a_2)) = 0.13 \times 101300 + (-3000) = 10169$$

$$g_1(a_6) - g_1(a2) = 103600 - 101300 = 2300$$

$$\therefore\ g_1(a_6) - g_1(a_2) < q_1(g_1(a_2)) \Longrightarrow \boldsymbol{a_2\ I_1\ a_6} \Rightarrow n_i(a_2, a_6) := 1$$

Table 1.16: Comparison of actions a_2 and a_6

	g_1: Prix $\downarrow\leftarrow$	g_2: Vmax $\uparrow\rightarrow$	g_3: C120 $\downarrow\rightarrow$	g_4: Coff $\uparrow\rightarrow$	g_5: Acce $\downarrow\rightarrow$	g_6: Frei $\downarrow\rightarrow$	g_7: Brui $\downarrow\rightarrow$
a_2: P205G	**101300**	205.3	7.9	**203**	**8.4**	78.3	73.4
a_6: RCLIO	103600	**205.1**	**8.2**	265	8.1	**81.7**	**73.6**

- On criterion g_2, the preferences are in increasing direction with the performances, and the thresholds are direct:

$$q_2(g_2(a_6)) = 0.02 \times 205.1 + 0 = 4.102$$

$$p_2(g_2(a_6)) = 0.05 \times 205.1 + 0 = 10.255$$

$$g_2(a_2) - g_2(a_6) = 205.3 - 205.1 = 0.2$$

$$\therefore\ g_2(a_2) - g_2(a_6) < q_2(g_2(a_6)) \Longrightarrow \boldsymbol{a_2\ I_2\ a_6} \Rightarrow n_i(a_2, a_6) := 1 + 1 = 2$$

- On criterion g_3, the preferences are in decreasing direction with the performances, and the thresholds are direct, and constant ($\alpha = 0$):

$$q_3(g_3(a_6)) = 1$$

$$p_3(g_3(a_6)) = 2$$

$$g_3(a_6) - g_3(a_2) = 8.2 - 7.9 = 0.3$$

$$\therefore\ g_3(a_6) - g_3(a_2) < q_3(g_3(a_6)) \Longrightarrow \boldsymbol{a_2\ I_3\ a_6} \Rightarrow n_i(a_2, a_6) := 2 + 1 = 3$$

- On criterion g_4, the preferences are in increasing direction with the performances, and the thresholds are direct, and constant ($\alpha = 0$):

$$q_4(g_4(a_2)) = 100$$

$$p_4(g_4(a_2)) = 200$$

$$g_4(a_6) - g_4(a_2) = 265 - 203 = 62$$

$$\therefore\ g_4(a_6) - g_4(a_2) < q_4(g_4(a_2)) \Longrightarrow \boldsymbol{a_2\ I_4\ a_6} \Rightarrow n_i(a_6, a_2) := 1$$

- On criterion g_5, the preferences are in decreasing direction with the performances, and the thresholds are direct:

$$q_5(g_5(a_2)) = 0.1 \times 8.4 + (-0.5) = 0.34$$

$$p_5(g_5(a_2)) = 0.2 \times 8.4 + (-1) = 0.68$$

$$g_5(a_2) - g_5(a_6) = 8.4 - 8.1 = 0.3$$

$$\therefore\ g_5(a_2) - g_5(a_6) < q_5(g_5(a_2)) \Longrightarrow \boldsymbol{a_2\ I_5\ a_6 \Rightarrow n_i(a_6, a_2) := 1 + 1 = 2}$$

- On criterion g_6, the preferences are in decreasing direction with the performances, and the thresholds are direct, and constant ($\alpha = 0$):

$$q_6(g_6(a_6)) = 0$$

$$p_6(g_6(a_6)) = 5$$

$$g_6(a_6) - g_6(a_2) = 81.7 - 78.3 = 3.4$$

$$\therefore\ q_6(g_6(a_6)) < g_6(a_6) - g_6(a_2) < p_6(g_6(a_6)) \Longrightarrow \boldsymbol{a_2\ Q_6\ a_6 \Rightarrow n_q(a_2, a_6) := 1}$$

- On criterion g_7, the preferences are in decreasing direction with the performances, and the thresholds are direct, and constant ($\alpha = 0$):

$$q_7(g_7(a_6)) = 3$$

$$p_7(g_7(a_6)) = 5$$

$$g_7(a_6) - g_7(a_2) = 73.6 - 73.4 = 0.2$$

$$\therefore\ g_7(a_6) - g_7(a_2) < q_7(g_7(a_6)) \Longrightarrow \boldsymbol{a_2\ I_7\ a_6 \Rightarrow n_i(a_2, a_6) := 3 + 1 = 4}$$

Obtaining consequently the following indicators: $n_p(a_2, a_6) = 0$, $n_q(a_2, a_6) = 1$, $n_i(a_2, a_6) = 4$, $n_i(a_6, a_2) = 2$, $n_q(a_6, a_2) = 0$, and $n_p(a_6, a_2) = 0$.

Since $n_p(a_6, a_2) + n_q(a_6, a_2) = 0$ and $n_i(a_6, a_2) < n_i(a_2, a_6) + n_q(a_2, a_6) + n_p(a_2, a_6)$ then, $\boldsymbol{a_2\ S_q\ a_6}$. Since $n_p(a_2, a_6) = 0$ then, $\boldsymbol{a_6\ S_s\ a_2}$. Since $n_q(a_2, a_6) > n_q(a_6, a_2) + n_p(a_6, a_2)$ then, $\boldsymbol{\sim (a_6\ S_s\ a_2)}$. Thus, $\sigma(a_2, a_6) = 1$, and $\sigma(a_6, a_2) = 0.4$.

For all pairs of actions representing the illustrative example, the credibility matrix is obtained for the fuzzy outranking relations (Table 1.17). This matrix can also be found in the next chapter (Figure 2.39 on page 96).

Table 1.17: Fuzzy outranking matrix in ELECTRE IV

$\sigma(.,.)$	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
a_1	1	0	0	0	0	0	0	0	0	0
a_2	0.8	1	0	0	0	1	0	0	0	0
a_3	0	0	1	0	0	0	0	0	0	0
a_4	0	0	0	1	0	0	0	0	0	0
a_5	0	0	0	0	1	0	0	0	0	0
a_6	0.6	0.4	0	0	0	1	0	0	0	0
a_7	1	0	0	0	0.2	0	1	0	0	0
a_8	0	0	0.4	0	0	0	0	1	0.8	0
a_9	0	0	0	0	0	0	0	0	1	0
a_{10}	0	0	0	0	0	0	0	0	0	1

Ranking algorithm

According to the credibility matrix obtaining by ELECTRE III (Table 1.13 on page 32, or Figure 2.33 on page 91) the ranking algorithm can be applied, in the case of **descending distillation**, as follows [68, p. 131-146]:

Let $\overline{A}_0 = A = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}\}$; $s(\lambda_k) = \alpha \times \lambda_k + \beta = -0.15 \times \lambda + 0.30$.

Distillation 1

Step 1: Let $k = 0$, $D_0 = A$ then, $\lambda_0 = \max\limits_{\substack{a,\, b \,\in \overline{A}_0 \\ a \neq b}} \sigma(a,b) \; = 1$, and

$\lambda_1 = \max\limits_{\substack{\{\sigma(a,b) > \lambda_0 - s(\lambda_0)\} \\ a,\, b \,\in D_0}} \sigma(a,b) = 0.846.$

Thus, $\lambda_0 - s(\lambda_0) = 1 - (-0.15 \times \lambda_0 + 0.30) = 0.85$, and 0.846 is the richest credibility degree lower than 0.85.

For the first step of the first distillation, $a\, S_{D_0}^{\lambda_1}\, b$ if and only if $\sigma(a,b) > \lambda_1 \Leftrightarrow \sigma(a,b) > 0.846$, and $\sigma(a,b) > \sigma(b,a) + s(\sigma(a,b)) \Leftrightarrow \sigma(a,b) > \sigma(b,a) + (-0.15 \times \sigma(a,b))$. The crispy outranking matrix is obtained (Table 1.18).

Thus, for each action, the calculation of the power, the weakness, and the qualification are presented in Table 1.19. The maximum λ_0-qualification is 1, then $\overline{D}_1 = \{a_2, a_6, a_7, a_8\}$.

Table 1.18: Crispy outranking matrix (Dist 1, Step 1)

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
a_1	0	0	0	0	0	0	0	0	0	0
a_2	1	0	0	0	0	0	0	0	0	0
a_3	0	0	0	0	0	0	0	0	0	0
a_4	0	0	0	0	0	0	0	0	0	0
a_5	0	0	0	0	0	0	0	0	0	0
a_6	1	0	0	0	0	0	0	0	0	0
a_7	1	0	0	0	0	0	0	0	0	0
a_8	0	0	0	0	0	0	0	0	1	0
a_9	0	0	0	0	0	0	0	0	0	0
a_{10}	0	0	0	0	0	0	0	0	0	0

Table 1.19: Power, weakness, and qualification (Dist 1, Step 1)

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
λ_0-power	0	1	0	0	0	1	1	1	0	0
λ_0-weakness	3	0	0	0	0	0	0	0	1	0
λ_0-qualification	-3	**1**	0	0	0	**1**	**1**	**1**	-1	0

Step 2: Let $k = 1$, $D_1 = \overline{D}_1 = \{a_2, a_6, a_7, a_8\}$. Then, the fuzzy outranking matrix is defined only for actions in D_1, as follows: From the first step, $\lambda_1 = 0.846$. Then,

Table 1.20: Credibility matrix (Dist 1, Step 2)

$\sigma(.,.)$	a_2	a_6	a_7	a_8
a_2	1	1	0.748	0.659
a_6	0.895	1	0.714	0.686
a_7	0.846	0.846	1	0.757
a_8	0.723	0.769	0.416	1

$\lambda_1 - s(\lambda_1) = 0.846 - (-0.15 \times \lambda_1 + 0.30) = 0.6729.$

Thus, $\lambda_2 = \max\limits_{\substack{\{\sigma(a,b) > \lambda_1 - s(\lambda_1)\} \\ a,\, b \in D_1}} \sigma(a,b) = 0.416.$

For the second step of the first distillation, $a\, S_{D_1}^{\lambda_2}\, b$ if and only if $\sigma(a,b) > \lambda_k \Leftrightarrow \sigma(a,b) > 0.416$, and $\sigma(a,b) > \sigma(b,a) + s(\sigma(a,b)) \Leftrightarrow \sigma(a,b) > \sigma(b,a) + (-0.15 \times \sigma(a,b))$. The crispy outranking matrix is obtained (Table 1.21).

Thus, for each action, the calculation of the power, the weakness, and the qualification are presented in Table 1.22. The maximum λ_1-qualification is 1. The only action that

Table 1.21: Crispy outranking matrix (Dist 1, Step 2)

	a_2	a_6	a_7	a_8
a_2	0	0	0	0
a_6	0	0	0	0
a_7	0	0	0	1
a_8	0	0	0	0

has the maximum qualification is a_7, then $\overline{C}_1 = \{a_7\}$. It means that a_7 is the best action according to the descending distillation.

Table 1.22: Power, weakness, and qualification (Dist 1, Step 2)

	a_2	a_6	a_7	a_8
λ_1-power	0	0	1	0
λ_1-weakness	0	0	0	1
λ_1-qualification	0	0	1	-1

For the next distillation, $\overline{A}_1 = \overline{A}_0 \setminus \overline{C}_1 = \{a_1, a_2, a_3, a_4, a_5, a_6, a_8, a_9, a_{10}\}$.

Distillation 2

Step 1: The fuzzy outranking matrix is now defined, only for actions in $\overline{A}_1$, in Table 1.23.

Table 1.23: Credibility matrix (Dist 2, Step 1)

$\sigma(.,.)$	a_1	a_2	a_3	a_4	a_5	a_6	a_8	a_9	a_{10}
a_1	1	0.692	0.692	0	0.153	0.692	0.382	0.562	0.382
a_2	0.898	1	0.734	0.163	0.623	1	0.659	0.692	0.735
a_3	0.769	0.671	1	0	0	0.769	0.846	0.71	0.692
a_4	0	0	0.392	1	0	0	0.647	0.923	0.846
a_5	0.396	0	0	0	1	0	0	0	0
a_6	0.965	0.895	0.815	0	0.525	1	0.686	0.769	0.692
a_8	0.665	0.723	0.843	0.767	0	0.769	1	0.963	0.846
a_9	0	0	0.32	0.769	0	0	0.484	1	0.822
a_{10}	0	0	0	0.355	0	0	0	0.769	1

Let $k = 0$, $D_0 = \overline{A}_1 = \{a_1, a_2, a_3, a_4, a_5, a_6, a_8, a_9, a_{10}\}$.

Then, $\lambda_0 = \max_{\substack{a,\, b \in \overline{D}_0 \\ a \neq b}} \sigma(a,b) = 1$, and

$$\lambda_1 = \max_{\substack{\{\sigma(a,b) > \lambda_0 - s(\lambda_0)\} \\ a,\, b \in D_0}} \sigma(a,b) = 0.846.$$

For the first step of the second distillation, $a\, S_{D_0}^{\lambda_1}\, b$ if and only if $\sigma(a,b) > \lambda_1 \Leftrightarrow \sigma(a,b) > 0.846$, and $\sigma(a,b) > \sigma(b,a) + s(\sigma(a,b)) \Leftrightarrow \sigma(a,b) > \sigma(b,a) + (-0.15 \times \sigma(a,b))$. The crispy outranking matrix is defined in Table 1.24.

Table 1.24: Crispy outranking matrix (Dist 2, Step 1)

	a_1	a_2	a_3	a_4	a_5	a_6	a_8	a_9	a_{10}
a_1	0	0	0	0	0	0	0	0	0
a_2	1	0	0	0	0	0	0	0	0
a_3	0	0	0	0	0	0	0	0	0
a_4	0	0	0	0	0	0	0	0	0
a_5	0	0	0	0	0	0	0	0	0
a_6	1	0	0	0	0	0	0	0	0
a_8	0	0	0	0	0	0	0	1	0
a_9	0	0	0	0	0	0	0	0	0
a_{10}	0	0	0	0	0	0	0	0	0

Thus, for each action, the calculation of the power, the weakness, and the qualification are presented in Table 1.25. The maximum λ_0-qualification is 1, then $\overline{D}_1 = \{a_2, a_6, a_8\}$.

Table 1.25: Power, weakness, and qualification (Dist 2, Step 1)

	a_1	a_2	a_3	a_4	a_5	a_6	a_8	a_9	a_{10}
λ_0-power	0	1	0	0	0	1	1	0	0
λ_0-weakness	2	0	0	0	0	0	0	1	0
λ_0-qualification	-2	**1**	0	0	0	**1**	**1**	-1	0

Step 2: Let $k = 1, D_1 = \overline{D}_1 = \{a_2, a_6, a_8\}$. Then, the fuzzy outranking matrix is defined only for actions in D_1 (Table 1.26).

From the first step, $\lambda_1 = 0.846$. Then, $\lambda_1 - s(\lambda_1) = 0.846 - (-0.15 \times \lambda_1 + 0.30) = 0.6729$.

Thus, $\lambda_2 = 0$ because $\forall\, a, b \in D_1 \sigma(a,b) > 0.6729$.

Table 1.26: Credibility matrix (Dist 2, Step 2)

$\sigma(.,.)$	a_2	a_6	a_8
a_2	1	1	0.659
a_6	0.895	1	0.686
a_8	0.723	0.769	1

Table 1.27: Crispy outranking matrix (Dist 2, Step 2)

	a_2	a_6	a_8
a_2	0	0	0
a_6	0	0	0
a_8	0	0	0

For the second step of the second distillation, the crispy outranking matrix is defined in Table 1.27.

Thus, for each action, the calculation of the power, the weakness, and the qualification are presented in Table 1.28. The maximum λ_1-qualification is 1. All the actions have the same qualification then, the second equivalence class is $\overline{C}_2 = \{a_2, a_6, a_8\}$.

Table 1.28: Power, weakness, and qualification (Dist 2, Step 2)

	a_2	a_6	a_8
λ_1-power	0	0	0
λ_1-weakness	0	0	0
λ_1-qualification	0	0	0

For the next distillation, $\overline{A}_2 = \overline{A}_1 \setminus \overline{C}_2 = \{a_1, a_3, a_4, a_5, a_9, a_{10}\}$.

Distillation 3

Step 1: The fuzzy outranking matrix is now defined, only for actions in $\overline{A}_2$, in Table 1.29.

Let $k = 0$, $D_0 = \overline{A}_2 = \{a_1, a_3, a_4, a_5, a_9, a_{10}\}$.

Then, $\lambda_0 = \max\limits_{\substack{a,\, b \,\in\, \overline{D}_0 \\ a \neq b}} \sigma(a,b) = 0.923$, and

$$\lambda_1 = \max_{\substack{\{\sigma(a,b) > \lambda_0 - s(\lambda_0)\} \\ a,\, b \,\in\, D_0}} \sigma(a,b) = 0.71.$$

Table 1.29: Credibility matrix (Dist 3, Step 1)

$\sigma(.,.)$	a_1	a_3	a_4	a_5	a_9	a_{10}
a_1	1	0.692	0	0.153	0.562	0.382
a_3	0.769	1	0	0	0.71	0.692
a_4	0	0.392	1	0	0.923	0.846
a_5	0.396	0	0	1	0	0
a_9	0	0.32	0.769	0	1	0.822
a_{10}	0	0	0.355	0	0.769	1

For the first step of the third distillation, $a\ S_{D_0}^{\lambda_1}\ b$ if and only if $\sigma(a,b) > \lambda_1 \Leftrightarrow \sigma(a,b) > 0.71$, and $\sigma(a,b) > \sigma(b,a) + s(\sigma(a,b)) \Leftrightarrow \sigma(a,b) > \sigma(b,a) + (-0.15 \times \sigma(a,b))$. The crispy outranking matrix is defined in Table 1.30.

Table 1.30: Crispy outranking matrix (Dist 3, Step 1)

	a_1	a_3	a_4	a_5	a_9	a_{10}
a_1	0	0	0	0	0	0
a_3	0	0	0	0	0	0
a_4	0	0	0	0	0	1
a_5	0	0	0	0	0	0
a_9	0	0	0	0	0	0
a_{10}	0	0	0	0	0	0

Thus, for each action, the calculation of the power, the weakness, and the qualification are presented in Table 1.31. The maximum λ_0-qualification is 1. The only action that has the maximum qualification is a_4, then $\overline{C}_3 = \{a_4\}$.

Table 1.31: Power, weakness, and qualification (Dist 3, Step 1)

	a_1	a_3	a_4	a_5	a_9	a_{10}
λ_0-power	0	0	1	0	0	0
λ_0-weakness	0	0	0	0	0	1
λ_0-qualification	0	0	**1**	0	0	-1

For the next distillation, $\overline{A}_3 = \overline{A}_2 \setminus \overline{C}_3 = \{a_1, a_3, a_5, a_9, a_{10}\}$.

Distillation 4

Step 1: The fuzzy outranking matrix is now defined, only for actions in $\overline{A}_3$, in Table 1.32.

Table 1.32: Credibility matrix (Dist 4, Step 1)

$\sigma(.,.)$	a_1	a_3	a_5	a_9	a_{10}
a_1	1	0.692	0.153	0.562	0.382
a_3	0.769	1	0	0.71	0.692
a_5	0.396	0	1	0	0
a_9	0	0.32	0	1	0.822
a_{10}	0	0	0	0.769	1

Let $k = 0$, $D_0 = \overline{A}_3 = \{a_1, a_3, a_5, a_9, a_{10}\}$.

Then, $\lambda_0 = \max\limits_{\substack{a,\,b\,\in\,\overline{D}_0 \\ a \neq b}} \sigma(a,b) \ = 0.822$, and

$\lambda_1 = \max\limits_{\substack{\{\sigma(a,b)\,>\,\lambda_0 - s(\lambda_0)\} \\ a,\,b\,\in\,D_0}} \sigma(a,b) = 0.562.$

For the first step of the fourth distillation, $a\, S_{D_0}^{\lambda_1}\, b$ if and only if $\sigma(a,b) > \lambda_1 \Leftrightarrow \sigma(a,b) > 0.562$, and $\sigma(a,b) > \sigma(b,a) + s(\sigma(a,b)) \Leftrightarrow \sigma(a,b) > \sigma(b,a) + (-0.15 \times \sigma(a,b))$. The crispy outranking matrix is defined in Table 1.33.

Table 1.33: Crispy outranking matrix (Dist 4, Step 1)

	a_1	a_3	a_5	a_9	a_{10}
a_1	0	0	0	0	0
a_3	0	0	0	1	1
a_5	0	0	0	0	0
a_9	0	0	0	0	0
a_{10}	0	0	0	0	0

Thus, for each action, the calculation of the power, the weakness, and the qualification are presented in Table 1.31. The maximum λ_0-qualification is 2. The only action that has the maximum qualification is a_3, then $\overline{C}_4 = \{a_3\}$.

For the next distillation, $\overline{A}_4 = \overline{A}_3 \setminus \overline{C}_4 = \{a_1, a_5, a_9, a_{10}\}$.

Table 1.34: Power, weakness, and qualification (Dist 4, Step 1)

	a_1	a_3	a_5	a_9	a_{10}
λ_0-power	0	2	0	0	0
λ_0-weakness	0	0	0	1	1
λ_0-qualification	0	**2**	0	-1	-1

Distillation 5

Step 1: The fuzzy outranking matrix is now defined, only for actions in $\overline{A}_4$, in Table 1.35.

Table 1.35: Credibility matrix (Dist 5, Step 1)

$\sigma(.,.)$	a_1	a_5	a_9	a_{10}
a_1	1	0.153	0.562	0.382
a_5	0.396	1	0	0
a_9	0	0	1	0.822
a_{10}	0	0	0.769	1

Let $k = 0$, $D_0 = \overline{A}_4 = \{a_1, a_5, a_9, a_{10}\}$.

Then, $\lambda_0 = \max\limits_{\substack{a,\, b \in \overline{D}_0 \\ a \neq b}} \sigma(a,b) = 0.822$, and

$$\lambda_1 = \max_{\substack{\{\sigma(a,b) > \lambda_0 - s(\lambda_0)\} \\ a,\, b \in D_0}} \sigma(a,b) = 0.562.$$

For the first step of the fifth distillation, $a\, S_{D_0}^{\lambda_1}\, b$ if and only if $\sigma(a,b) > \lambda_1 \Leftrightarrow \sigma(a,b) > 0.562$, and $\sigma(a,b) > \sigma(b,a) + s(\sigma(a,b)) \Leftrightarrow \sigma(a,b) > \sigma(b,a) + (-0.15 \times \sigma(a,b))$. The crispy outranking matrix is defined in Table 1.36.

Table 1.36: Crispy outranking matrix (Dist 5, Step 1)

	a_1	a_5	a_9	a_{10}
a_1	0	0	0	0
a_3	0	0	0	0
a_5	0	0	0	0
a_9	0	0	0	0
a_{10}	0	0	0	0

Thus, for each action, the calculation of the power, the weakness, and the qualification are presented in Table 1.31. The maximum λ_0-qualification is 0, then $\overline{D}_1 =$

$\{a_1, a_5, a_9, a_{10}\}$. Since $|\overline{D}_1| \neq 0$, and $\lambda_1 \neq 0$, then go to step 2.

Table 1.37: Power, weakness, and qualification (Dist 5, Step 1)

	a_1	a_5	a_9	a_{10}
λ_0-power	0	0	0	0
λ_0-weakness	0	0	0	0
λ_0-qualification	0	0	0	0

Step 2: Let $k = 1$, $D_1 = \overline{D}_1 = \{a_1, a_5, a_9, a_{10}\}$. Then, the fuzzy outranking matrix is the same that has been defined in Table 1.35.

From the first step, $\lambda_1 = 0.562$. Then, $\lambda_1 - s(\lambda_1) = 0.562 - (-0.15 \times \lambda_1 + 0.30) = 0.3463$.

Thus, $\lambda_2 = \max_{\substack{\{\sigma(a,b) > \lambda_1 - s(\lambda_1)\} \\ a,\, b \in D_1}} \sigma(a,b) = 0.153$.

For the second step of the fifth distillation, $a\, S_{D_1}^{\lambda_2}\, b$ if and only if $\sigma(a,b) > \lambda_k \Leftrightarrow \sigma(a,b) > 0.153$, and $\sigma(a,b) > \sigma(b,a) + s(\sigma(a,b)) \Leftrightarrow \sigma(a,b) > \sigma(b,a) + (-0.15 \times \sigma(a,b))$. The crispy outranking matrix is obtained (Table 1.38).

Table 1.38: Crispy outranking matrix (Dist 5, Step 2)

	a_1	a_5	a_9	a_{10}
a_1	0	0	1	1
a_5	1	0	0	0
a_9	0	0	0	0
a_{10}	0	0	0	0

Thus, for each action, the calculation of the power, the weakness, and the qualification are presented in Table 1.39. The maximum λ_1-qualification is 1. Two actions have the maximum qualification: a_1, and a_5 then, $\overline{D}_2 = \{a_1, a_5\}$. Since $|\overline{D}_2| \neq 0$, and $\lambda_2 \neq 0$ then, go to step 3.

Table 1.39: Power, weakness, and qualification (Dist 5, Step 2)

	a_1	a_5	a_9	a_{10}
λ_1-power	2	1	0	0
λ_1-weakness	1	0	1	1
λ_1-qualification	**1**	**1**	-1	-1

Step 3: The fuzzy outranking matrix is now defined, only for actions in $\overline{D}_2$ (Table 1.40).

Table 1.40: Credibility matrix (Dist 5, Step 3)

$\sigma(.,.)$	a_1	a_5
a_1	1	0.153
a_5	0.396	1

From the second step, $\lambda_2 = 0.153$. Then, $\lambda_2 - s(\lambda_2) = 0.153 - (-0.15 \times \lambda_2 + 0.30) = -0.12405$.

Thus, $\lambda_3 = 0$ because $\forall\, a, b \in D_2 \sigma(a,b) > -0.12405$.

For the third step of the fifth distillation, $a\, S_{D_0}^{\lambda_1}\, b$ if and only if $\sigma(a,b) > \lambda_1 \Leftrightarrow \sigma(a,b) > 0.153$, and $\sigma(a,b) > \sigma(b,a) + s(\sigma(a,b)) \Leftrightarrow \sigma(a,b) > \sigma(b,a) + (-0.15 \times \sigma(a,b))$. The crispy outranking matrix is defined in Table 1.41.

Table 1.41: Crispy outranking matrix (Dist 5, Step 3)

	a_1	a_5
a_1	0	0
a_5	1	0

Thus, for each action, the calculation of the power, the weakness, and the qualification are presented in Table 1.42. The maximum λ_0-qualification is 1. The only action that has the maximum qualification is a_5, then $\overline{C}_5 = \{a_5\}$.

Table 1.42: Power, weakness, and qualification (Dist 5, Step 3)

	a_1	a_5
λ_2-power	0	1
λ_2-weakness	1	0
λ_2-qualification	-1	1

For the next distillation, $\overline{A}_5 = \overline{A}_4 \setminus \overline{C}_4 = \{a_1, a_9, a_{10}\}$.

Distillation 6

Step 1: The fuzzy outranking matrix is now defined, only for actions in $\overline{A}_5$ (Table 1.43).

Table 1.43: Credibility matrix (Dist 6, Step 1)

$\sigma(.,.)$	a_1	a_9	a_{10}
a_1	1	0.562	0.382
a_9	0	1	0.822
a_{10}	0	0.769	1

Let $k = 0$, $D_0 = \overline{A}_5 = \{a_1, a_9, a_{10}\}$.

Then, $\lambda_0 = \max\limits_{\substack{a,\, b\, \in \overline{D}_0 \\ a \neq b}} \sigma(a,b) \; = 0.822$, and

$\lambda_1 = \max\limits_{\substack{\{\sigma(a,b) > \lambda_0 - s(\lambda_0)\} \\ a,\, b\, \in D_0}} \sigma(a,b) = 0.562.$

For the first step of the sixth distillation, $a\, S_{D_0}^{\lambda_1}\, b$ if and only if $\sigma(a,b) > \lambda_1 \Leftrightarrow \sigma(a,b) > 0.562$, and $\sigma(a,b) > \sigma(b,a) + s(\sigma(a,b)) \Leftrightarrow \sigma(a,b) > \sigma(b,a) + (-0.15 \times \sigma(a,b))$. The crispy outranking matrix is defined in Table 1.44.

Table 1.44: Crispy outranking matrix (Dist 6, Step 1)

	a_1	a_9	a_{10}
a_1	0	0	0
a_9	0	0	0
a_{10}	0	0	0

Thus, for each action, the calculation of the power, the weakness, and the qualification are presented in Table 1.45. The maximum λ_0-qualification is 0, then $\overline{D}_1 = \{a_1, a_9, a_{10}\}$. Since $|\overline{D}_1| \neq 0$, and $\lambda_1 \neq 0$, then go to step 2.

Table 1.45: Power, weakness, and qualification (Dist 6, Step 1)

	a_1	a_9	a_{10}
λ_0-power	0	0	0
λ_0-weakness	0	0	0
λ_0-qualification	0	0	0

Step 2: Let $k = 1$, $D_1 = \overline{D}_1 = \{a_1, a_9, a_{10}\}$. Then, the fuzzy outranking matrix is the same that has been defined in Table 1.43.

From the first step, $\lambda_1 = 0.562$. Then, $\lambda_1 - s(\lambda_1) = 0.562 - (-0.15 \times \lambda_1 + 0.30) = 0.3463$.

Thus, $\lambda_2 = 0$, and the crispy outranking matrix is obtained (Table 1.46).

Table 1.46: Crispy outranking matrix (Dist 6, Step 2)

	a_1	a_9	a_{10}
a_1	0	1	1
a_9	0	0	0
a_{10}	0	0	0

Thus, for each action, the calculation of the power, the weakness, and the qualification are presented in Table 1.39. The maximum λ_1-qualification is 2. The only action that has the maximum qualification is a_1, then $\overline{C}_6 = \{a_1\}$.

Table 1.47: Power, weakness, and qualification (Dist 6, Step 2)

	a_1	a_9	a_{10}
λ_1-power	2	0	0
λ_1-weakness	0	1	1
λ_1-qualification	**2**	-1	-1

For the next and last distillation, $\overline{A}_6 = \overline{A}_5 \setminus \overline{C}_5 = \{a_9, a_{10}\}$.

Distillation 7

Step 1: The fuzzy outranking matrix is now defined, only for actions in $\overline{A}_6$ (Table 1.48).

Table 1.48: Credibility matrix (Dist 7, Step 1)

$\sigma(.,.)$	a_9	a_{10}
a_9	1	0.822
a_{10}	0.769	1

Let $k = 0$, $D_0 = \overline{A}_6 = \{a_9, a_{10}\}$.

Then, $\lambda_0 = \max\limits_{\substack{a,\,b\,\in\,\overline{D}_0 \\ a\neq b}} \sigma(a,b) = 0.822$, and $\lambda_1 = 0$

The crispy outranking matrix is defined in Table 1.49.

Table 1.49: Crispy outranking matrix (Dist 7, Step 1)

	a_9	a_{10}
a_9	0	0
a_{10}	0	0

Thus, for each action, the calculation of the power, the weakness, and the qualification are presented in Table 1.50. The maximum λ_0-qualification is 0. The two remaining actions have the maximum qualification then $\overline{C}_7 = \{a_9, a_{10}\}$.

Table 1.50: Power, weakness, and qualification (Dist 7, Step 1)

	a_9	a_{10}
λ_0-power	0	0
λ_0-weakness	0	0
λ_0-qualification	0	0

At the end of the seventh distillation all the actions are ranked, then END of the distillation process.

The Table 1.51 presents a resume of the different ranking algorithm calculations. In the first rank of the final pre-order there is only the action a_7 (R21TS) which belongs to the first class of the median pre-order. In the second rank, the actions a_2 (P205G), and a_6 (RCLIO) are indifferent and placing in the same class on the median pre-order. In the third rank, two actions are incomparable: a_4 (P605S), and a_8 (R21TU) which must be distinguished. The action a_4 is placed in the fifth position on the descending distillation, and in the fourth position on the ascending distillation, with 1 point of difference from the two positions. The action a_8 is placed in the second position on the descending distillation, and in the sixth position on the ascending distillation, with 4 points of difference from the two positions. Thus, a_8 is better ranked on the descending distillation than on the ascending distillation. The same analysis can be made for all the actions on the partial pre-order.

The complete results such as the two distillations results (Figure 2.29 on page 87), the final ranks (Figure 2.30 on page 88), the median pre-order (Figure 2.31 on page 89), the ranking

Table 1.51: Resume of the ranking results

	Ranks in the final pre-order	Position in the descending distillation	Position in the ascending distillation	Position differences	Ranks in the median pre-order
a_7: R21TS	1	1	1	0	1
a_2: P205G	2	2	1	1	2
a_6: RCLIO	2	2	1	1	2
a_4: P605S	3	5	4	1	3
a_8: R21TU	3	2	6	4	4
a_3: P405M	4	6	6	0	5
a_5: R4GTL	4	7	5	2	6
a_{10}: ALPIN	5	9	8	1	7
a_1: CBX16	5	8	10	2	8
a_9: R25BA	6	9	9	0	9

matrix (e.g. Figure 2.35 on page 92), and the final graph (Figure 2.36 on page 93), according to the illustrative example, can be found in the next chapter, which presents the ELECTRE III-IV software.

Chapter 2

The Software ELECTRE III-IV

The main objective of this chapter is to present the ELECTRE III-IV software, with the help of the interface, such as the ones corresponding to the input data, the calculation, and the interpretation of the results.

After the progress of the calculations, a **graph** is built representing the partial pre-order: all of the actions are ranked from the best to the worst. The transitivity arcs are omitted. The comparison between two given actions *a* and *b* can lead to four different cases:

- *a* may be better than *b* (graphically, there are consecutive arcs from action *a* to action *b*).
- *b* may be better than *a* (graphically, there are consecutive arcs from action *b* to action *a*).
- *a* and *b* may be indifferent (graphically, they are displayed in the same box).
- *a* and *b* may be incomparable (graphically, they are not connected by any arc or a succession of consecutive arcs).

If the user wants to see the progress of the calculation, he/she can visualize the following results:

1) **The distillations results**: two contiguous lists of ranked actions for the two distillations.

2) **The ranks in final pre-order**: only the ranks of the actions in the final pre-order are displayed, without the incomparabilities.

3) **The median pre-order**: a complete pre-order (i.e. any two actions are comparable: one being better than the other or the two being equivalent) built from the final partial preorder. This ranking is another possibility for the users who do not wish to take into account incomparabilities.

4) **The ranking matrix**: the matrix of the final pre-order which offers a synthetic view of the results. In the intersection of the line corresponding to the action *a* and the column corresponding to action *b*, the following interpretations may be done:

   . **P** or $\succ$ if *a* is better than *b*, in one of the pre-orders and at least as well ranked in the other pre-order.

   . **I** or $\equiv$ if *a* is equivalent to *b*.

   . $\mathbf{P}^{-}$ or $\prec$ if *a* is ranked worst than *b* in one of the pre-orders and at least as well ranked in the other pre-order.

   . **R** or a green square $\square$ if *a* is incomparable to *b*.

After some advice of installation on the your computer, this chapter proposes a set of screen copies that has the goal to guide the user in different stages of the conception, and the exploration of the project data.

2.1 Configuration and requirements

ELECTRE III-IV software, version 3.x, was developed with Borland C++ programming language using the Microsoft Windows interface, by Pitor ZIELNIEWICZ (Institute of Computing Science of the Poznan University of Technology) under the supervision of Professors Bernard ROY and Roman SŁOWIŃSKI.

This software runs on the following operating systems: Windows 3.1, 95, 98, 2000, Me and XP. In order to instal ELECTRE III-IV, version 3.1b, you must run the `install.exe` setup program. On contrary, to install the version 3.1a, run the `instal.bat` setup program.

2.2 Interface structure of the software

This section presents the interface structure of the ELECTRE III-IV software. To start a session, just click in the icon **ELECTRE**. The main window is composed by seven top-down menus: File, Edit, Calculate, Results, Options, Window, and Help (Figure 2.1).

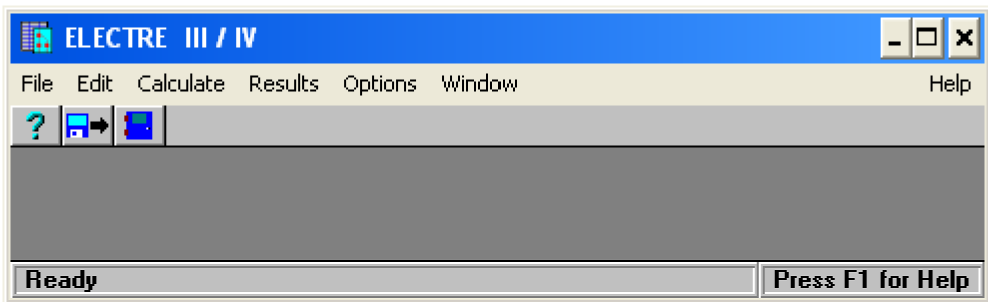


Figure 2.1: Main menu

In the beginning of the session, the *toolbar* propose three buttons: help, open a project, and exit the software (from the left to the right). In the bottom of the window, the *status bar* display some useful information about the option selected. Both the toolbar and the *status bar* can be visible or invisible by selecting the right option in the **Options** menu.

All of the options are accessible just using the keyboard by pressing the key **ALT** and without releasing it, also press the key corresponding to the letter underlined in the name of the option (for instance, to activate the **File** menu, press **ALT+F**). Use the move keys ($\uparrow$, $\downarrow$, $\rightarrow$, $\leftarrow$) to move between different selected options. The key **TAB** ($\leftrightarrows$) is used to move between different buttons, or boxes in the same window. To close a window, press **ALT+F4**.

The contents of the different standard menus is the following:

- **File menu** [**ALT+F**]: allows the user for the classic options of the menu file, such as create, open, or save a project.
- **Edit menu** [**ALT+E**]: enable the user to enter the data required by ELECTRE III and ELECTRE IV methods (project reference, criteria, actions, performances, and thresholds).
- **Calculate menu** [**ALT+C**]: allows the user to make the calculations, to choose the method to be used, and also statistics, and information about the current project.

- **Results menu** [ALT+R]: enable the user to display the results after the calculations process, such as the distillation result, the ranks in the final pre-order, the final graph, and also additional results such as the credibility matrix.

- **Options menu** [ALT+O]: allows the user to configure some preferences on the displaying results and tools menus.

- **Window menu** [ALT+W]: gives the possibility to organize the appearance of the windows on the screen.

- **Help menu** [ALT+H]: provides the user an online help, include in the software, and also the information version and licence of the software.

2.2.1 File menu

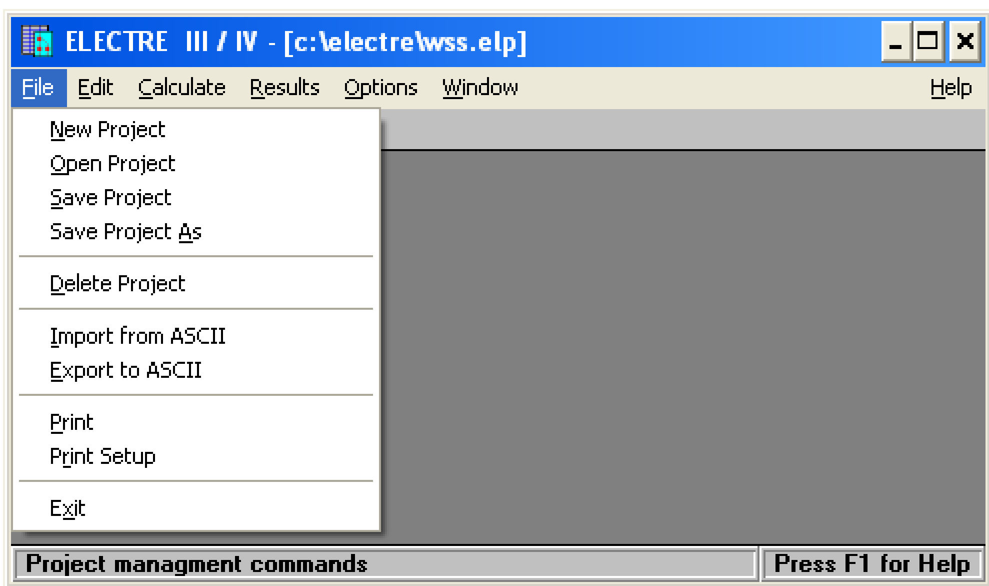


Figure 2.2: File menu

The following 10 options are available in the **File** menu (Figure 2.2):

- **New Project** [ALT+F+N]: allows to create a new project (Figure 2.3). This new project will be associated to an existing data set or a new data set, as follows:
  - **defining a new data set**: criteria, actions, performances, thresholds, or actions and the *matrix of degrees of credibility*. By selecting this option the Edit Project

Reference window (Figure 2.9, page 62) is available. Then it is necessary to use the options in the **Edit** menu (See section 2.2.2, page 60).

– **browsing an exiting data set**: this option will be used after the conversion of an old data set in a previous version of this software (Figure 2.4). The data conversion can be done using the MS-DOS `ELCNV.EXE` software that integrates this package.

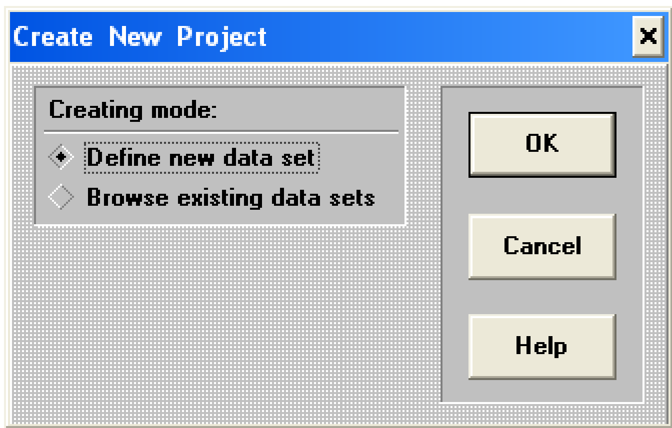


Figure 2.3: Create new project

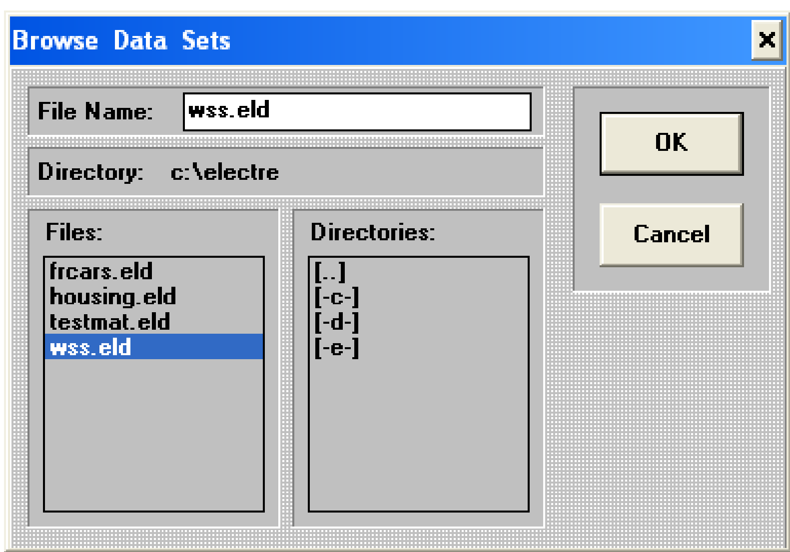


Figure 2.4: Browse data sets

A project is defined by a reference set of general information, including the name of the owner, notes, ... to be saved in a file with `*.elp` extension; and the main data set to be saved in a file with `*.eld` extension. This data set includes the family of pseudo-criteria, the set of actions, the table of performances.

- **Open Project** [ALT+F+O]: it may be used to load to the memory a data set created during a previous session of ELECTRE III-IV software and that has been saved on disk (or click in the second button in the toolbar from the left to the right). You have to type the name of the project or to select it in the file list. You may choose the device on which your file is saved and the directory in the Directories box. The Files box gives a list of all files that have the mask proposed in the File Name box. By default, ELECTRE III-IV software gives a list of the files having the extension `.elp` in the current Directory.

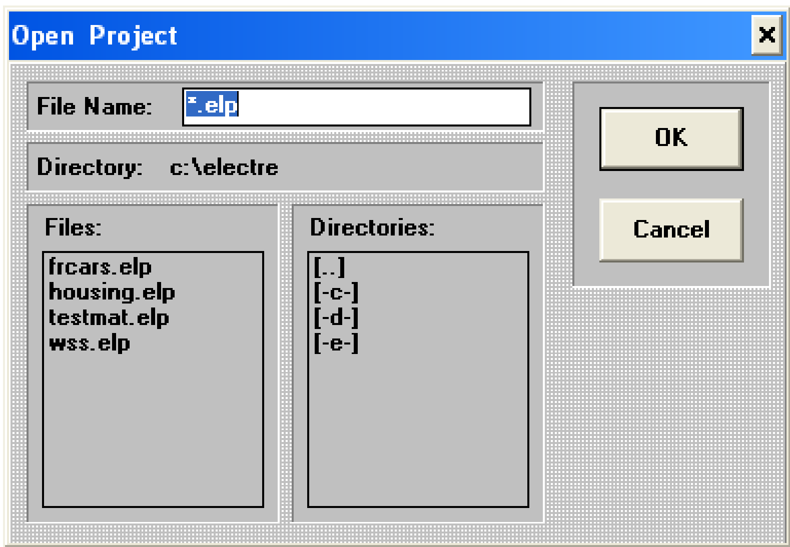


Figure 2.5: Open project

- **Save Project** [ALT+F+S]: allows to save the project currently in memory with its current name. It can only be used when a project has previously been created or loaded (or click in the third button in the toolbar from the left to the right). If the project has just been created, ELECTRE software displays the window Save Project As so that you may give a name to your file project.

- **Save Project As** [ALT+F+A]: allows to save the current project under a different name from its current name or to save a project for the first time. To save a project with its current name, you should use the option **File / Save Project**. Choose the device and the directory in the box directories and type the name of the file in the File Name box. If you do not give any extension to the file name, ELECTRE software will add the extension `*.elp`. Please note that, in any case, the associated data set will have `*.elp` extension (Figure 2.6). If you type an existing file name in the chosen directory, ELECTRE software will ask confirmation before removing the existing file.

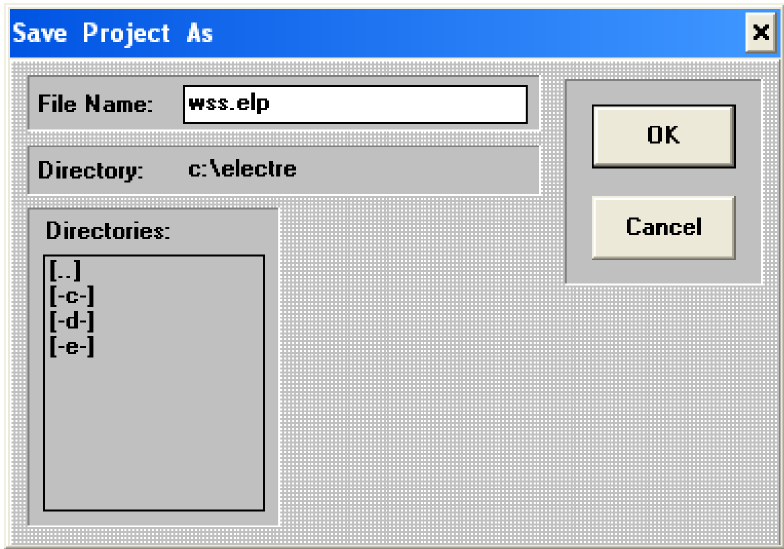


Figure 2.6: Save project

- **Delete Project** [ALT+F+D]: may be used to delete from the disk all the files related to a given project. ELECTRE software recalls the references of the project, then asks for confirmation before really removing the files.

- **Import from ASCII** [ALT+F+I]: allows to convert an `ASCII` file containing all the data characterizing a project (`*.elp` and `*.eld`) in data files for ELECTRE software. The required syntax for the project to be readable by ELECTRE software and to be converted is described in the file `README.ASC`. To read such a file, you need to convert an existing project in an `ASCII` file using the option **Export to ASCII**. You have to type in the dialog box the name of the file to be converted. By default, ELECTRE software gives a list of all the files that have the extension `*.txt` in the current directory. You may type the full name of the file (with device name and path) in the File Name box or modify the directory and the device in the Directories box and select a file name in the Files box.

- **Export to ASCII** [ALT+F+E]: may be used to convert the files related to a project in an `ASCII` file. The file syntax is explained in the file `README.ASC`.

- **Print** [ALT+F+P]: allows to print all or part of the data and/or all or part of the results (Figure 2.7). You have to select the elements you wish to print (or click in the fourth button in the toolbar from the left to the right). Use the **Print Setup** option to choose the printer and to define printing parameters. You may also print in a file (the default name of the project with the extension `*.prn` is proposed).

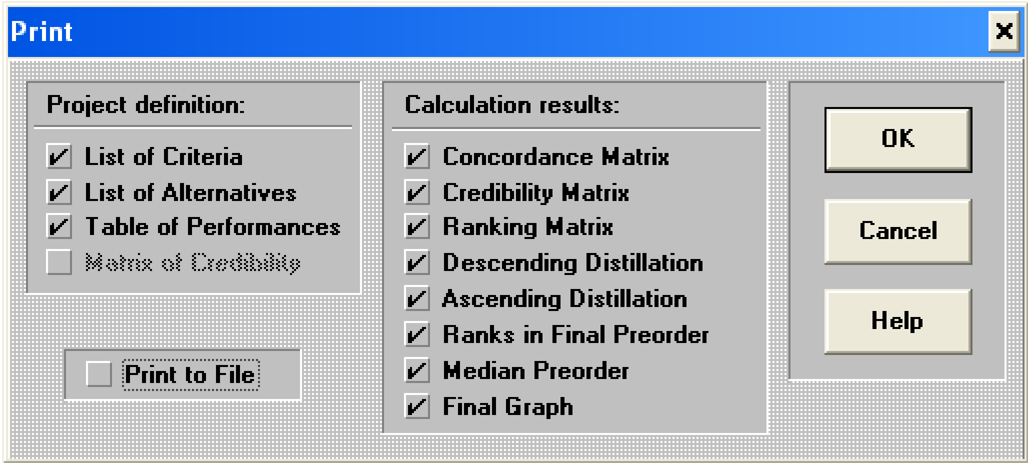


Figure 2.7: Print project

- **Print Setup** [ALT+F+R]: may be used to choose the printer and to define printing parameters such as the orientation (Portrait or Landscape). These parameters depend on the selected printer. The printers that are displayed are those installed on the computer.

- **Exit** [ALT+F+X]: closes ELECTRE software (or click in the last button in the toolbar from the left to the right). Use also the system box of the ELECTRE software. If the current project has been modified since it has last been saved on disk, ELECTRE software will ask if you would like to save before exiting.

2.2.2 Edit menu

For the **Edit** menu the following 6 options are available (Figure 2.8):

- **Project Reference** [ALT+E+R]: may be used to visualize and/or modify the references related to the project: owner's name, project description, type of data set (ELECTRE III, ELECTRE IV, *Matrix of degrees of credibility*).

- **Criteria** [ALT+E+C]: allows to display the list of criteria of the current project (name, code, direction of preferences, and weight if the project is of ELECTRE III type). It also allows to modify, insert, and delete a criterion. This option is not available if the project is of *Matrix of degrees of credibility* type.

- **Alternatives** [ALT+E+A]: allows to display the list of actions of the current project (name and code); and also to modify, insert, or delete actions.

- **Performances** [ALT+E+P]: may be used to type, display, and modify the table of performances of the actions of the family of pseudo-criteria. This option is not available if the project is of *Matrix of degrees of credibility* type.

- **Thresholds** [ALT+E+T]: may be used to type, display, and modify the values of the coefficients of the threshold functions of each criterion. This option is not available if the project is of *Matrix of degrees of credibility* type.

- **Matrix of Credibility** [ALT+E+M]: may be used to type, display, and modify the values of the credibility degrees of the matrix on which the user wishes to apply the ELECTRE III ranking algorithm. This option is only available if the project is of *Matrix of degrees of credibility* type.

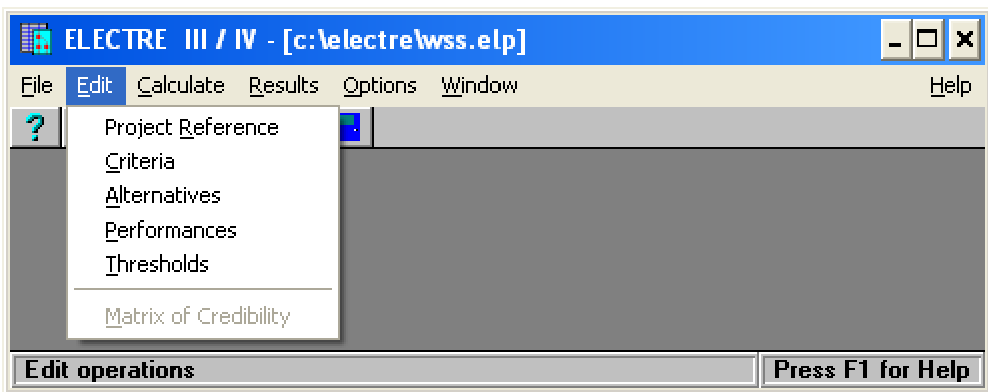


Figure 2.8: Edit menu

2.2.2.1 Edit Project Reference

The option **Edit / Project Reference** is used to type or modify some information on the project: owner's name (string with at most 40 characters); project description (string with at most 250 characters); and data set type (Figure 2.9).

There are three data set types. A project may be of both ELECTRE III and ELECTRE IV types. On the contrary, the type *Matrix of degrees of credibility* excludes the two others. When a data set belongs to the two types (ELECTRE III and ELECTRE IV), you have to choose the method to use before calculating. In this case, the software stores in memory and saves in a file the list of weights given to each criterion (ELECTRE III) and the list of chosen relations (ELECTRE IV). If a project is only given the ELECTRE IV type and that,

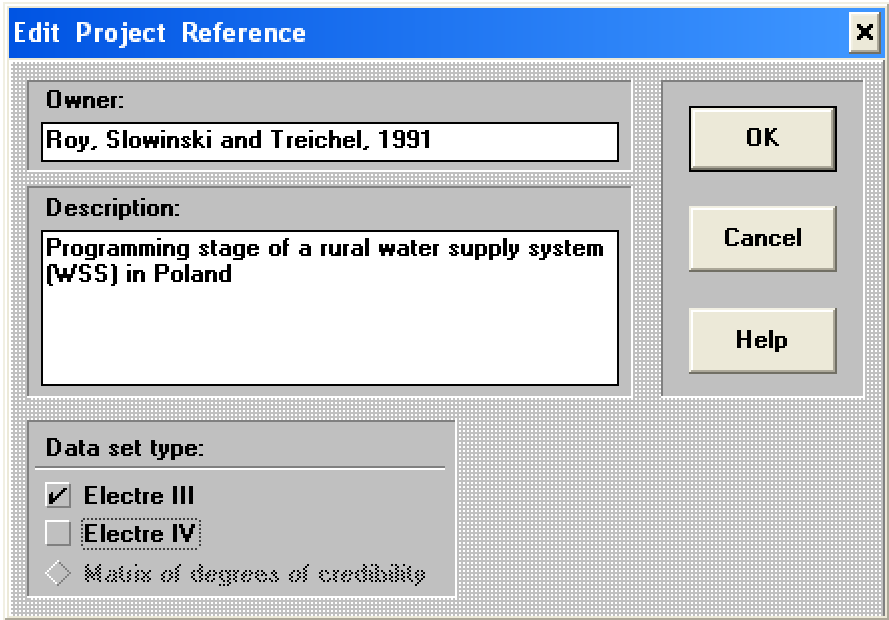


Figure 2.9: Edit project reference

afterwards, you would like to use ELECTRE III method, the software will ask you to define weights for the criteria. In the same way, if you change from ELECTRE III to ELECTRE IV you must choose a set of relations.

2.2.2.2 Edit Criteria

The option **Edit / Criteria** is used to define or modify a family of pseudo-criteria (Figure 2.10). In this window it is possible to see the number of criteria already defined for the current project (0 if it is a new project), the list of the codes for the criteria and the complementary information of each criterion. This information (name, code, weight, and direction of preference) is updated each time a new criterion is selected in the list of criteria.

The maximum number of criteria depends on the available memory of your computer (the use of these methods makes sense only with a minimum of three criteria). In the bottom of Figure 2.10 there are five buttons: Close, Modify, Insert, Delete, and Help:

- **Close**: closes the window and saves in the memory of the computer the modifications in the list of criteria. Beware, to save on the disk, the option **File / Save Project** must be selected.

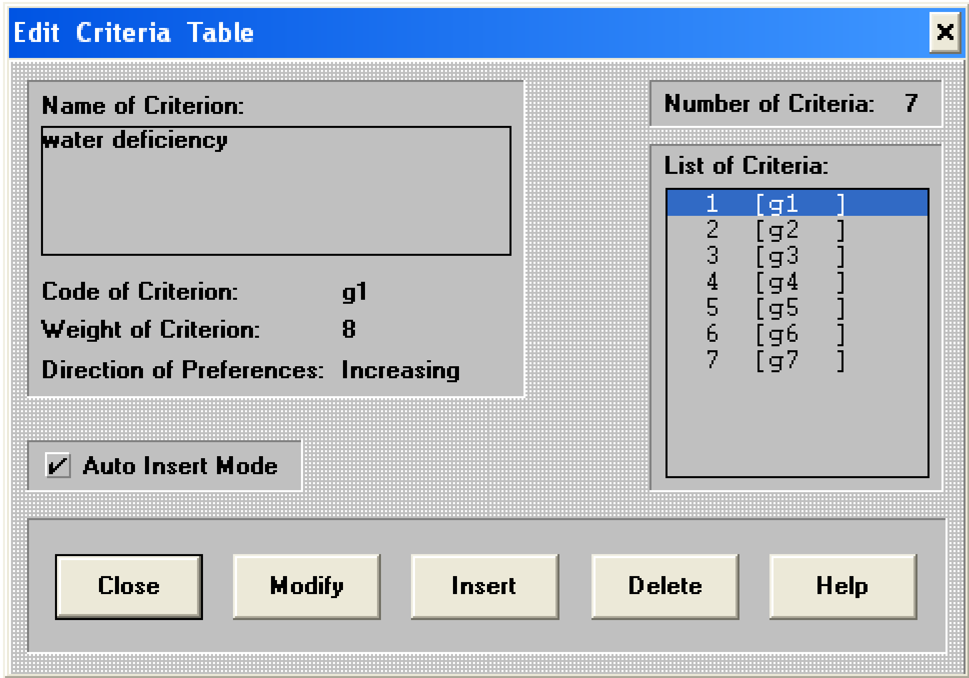


Figure 2.10: Edit criteria table

- **Modify**: allows modifications on the criterion that has been selected. The following two actions are equivalent:

  1) select a criterion in the list by clicking once on its code then click on the button Modify.

  2) double-click on the code of the criterion in the list of criteria.

  Then, the window Edit Criterion (Figure 2.11) is opened and you may modify the name, code, and/or the preference direction of the selected criterion.

- **Insert**: allows insertion of a new criterion which will be placed immediately after the current selected criterion. By clicking on the button, an empty window Edit Criterion is opened and you may define a new criterion; if the button **Auto Insert Mode** is activated, Edit Criterion empty windows, like Figure 2.11, will continuously appear until **Close** is clicked. This is a way to define criteria one after the another without having to go back to the Edit Criteria Table window (Figure 2.10), especially during the definition of a new project.

- **Delete**: may be used to delete the selected criteria. You will be asked for confirmation.

- **Help**: gives the user an online help for the contents of the current window.

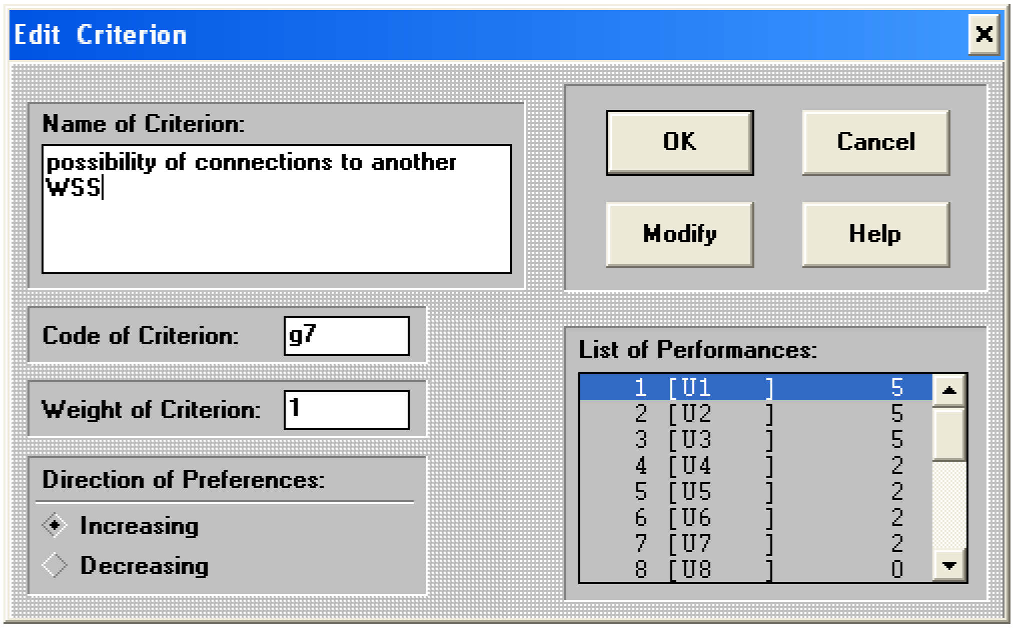


Figure 2.11: Edit criterion

The Figure 2.11 is obtained after the selection of the option **Edit / Criteria**, then of the button **Insert** or the button **Modify**. In the first case (Insert), you will obtain a window whose entry zones are empty, in the second case (Modify) the type zones will contain the characteristics of the criterion to be modified. A criterion is defined by a:

1) **Name**: you may enter any string with a maximum of 80 characters in the window for edition (null are permitted).

2) **Code**: consists of four alpha-numerical characters. Any letters or digits are authorized. If you do not wish to enter a code, the program will automatically associate to the criteria the next code not already defined in the list: Cr01, Cr02, Cr03 ...

3) **Weight**: if the project is of ELECTRE III type: this weight is an integer or decimal strictly greater than zero and smaller than or equal to 100. You may type at most five characters: you may therefore type three decimals for a number between 0 and 10, two decimals for a number greater than 10. In an ELECTRE III type project every criterion must have a weight. The weights are used during the aggregation of the indices of partial concordance (on each criterion) in a comprehensive concordance index.

4) **Direction of Preferences**: a criterion has an increasing preference direction if the greater values are preferred to the smaller ones; that is if the objective is to maximize

this criterion. On the contrary, a criterion has a decreasing preference direction if the smaller values are to be preferred (minimization is looking for).

If actions have already been defined, the Figure 2.11 will show the list of the codes of the actions and their performances on the criteria. It is possible to type or modify the performance of an action on the active criterion with this window (Figure 2.14). See above the button **Modify**. The window Edit Criterion (Figure 2.11) has four buttons: OK, Cancel, Modify, and Help:

- **OK**: closes the window Edit Criterion and stores in memory the characteristics entered for this criterion. The following events depend on the way you entered this option:

  1) if you entered this window by selecting the button **Modify** in the window Edit Criteria Table (Figure 2.10) or the button **Insert** of this window (Figure 2.11) with the button **Auto Insert Mode** not activated, activating the OK button brings you back to the window Edit Criteria Table (Figure 2.10). It is then possible to modify another criterion, to insert a criterion in another place in the list or to quit the window Edit Criteria Table (Figure 2.10).
  2) if you entered the window by selecting the button **Insert** in the window List of Criteria with the button **Auto Insert Mode** activated, activating the button OK displays a new window Edit Criterion (like Figure 2.11) with empty entry zones: you may then define a new criterion which will be inserted in the list after the selected one.

- **Cancel**: closes the window but does not keep in memory the possible modifications. It is the best way to stop an automatic insertion of criteria or to cancel the modifications that have just been made to a criterion.

- **Modify**: allows to modify or to enter the performance of an action on a criterion. The two following possibilities are equivalent:

  1) select an action in the list by clicking once on its code and click on the button Modify.
  2) double-click on the code of the action in the list.

  A window Edit Performance (Figure 2.14) will appear and you may modify the performance of the selected action on the current criterion.

- **Help**: gives the user an online help for the contents of the current window.

2.2.2.3 Edit Alternatives

The option **Edit / Alternatives**[1] allows definitions, or modifications of a set of actions. The actions are the objects to be compared: they may be solutions, plans, projects... The Figure 2.12 shows the "Number of Alternatives" (i.e. the number of actions) already defined for the project (0 if it is a new project), the "Name of Alternative" (i.e. the name of the selected action), and the "Code of Alternative" (i.e. the label of the selected action). This two last information is updated each time a new action is selected in the "List of Alternatives".

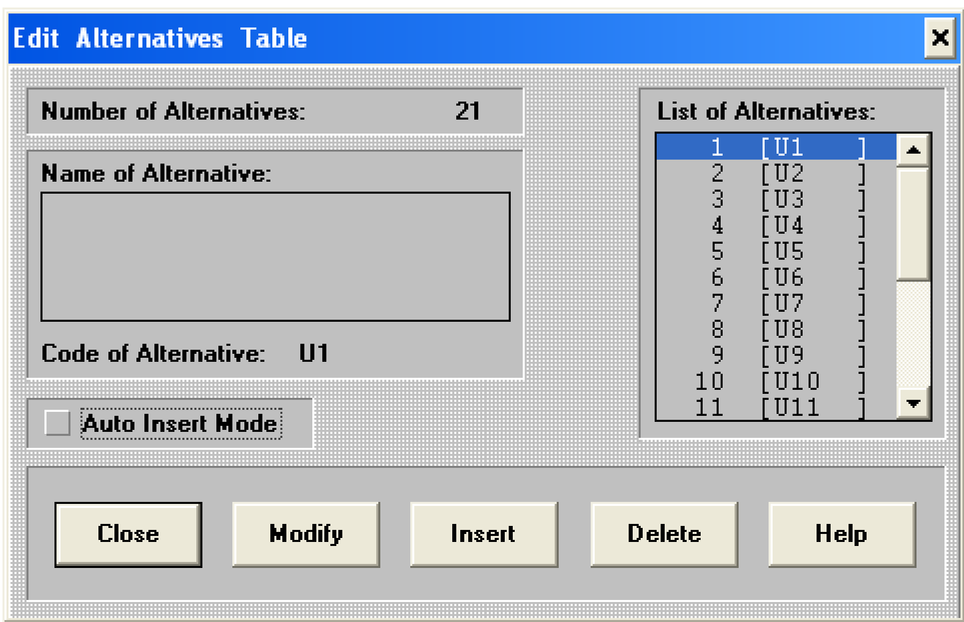


Figure 2.12: Edit alternatives table

The maximum number of actions depends on the available memory on your computer. In the bottom of Figure 2.12 there are five buttons: Close, Modify, Insert, Delete, and Help:

- **Close**: closes the window and stores in the memory of the computer the modifications of the list of actions. Beware, to save on the disk you need to select afterwards the option **File / Save Project**.

[1]The software can be used to rank a set actions that are not mutually incompatible when considering two distinct actions under analysis.

- **Modify**: modifies the selected action. The following two actions are equivalent:

  1) select an action in the list by clicking once on its code, then click on the button Modify.

  2) double-click on the code of the action in the list of actions.

  Then you may modify the name and/or the code of the selected action in the window Edit Alternative (Figure 2.13).

- **Insert**: allows for insertion of a new action which will be placed immediately after the activated action. By clicking on this button, an empty window Edit Alternative (like Figure 2.13) is opened and you may define a new action; if the button **Auto Insert Mode** is activated, empty windows Edit Alternative (like Figure 2.13) will be opened one after another until you select **Cancel** in one of these windows. This facility allows definitions of actions one after the other without having to come back to the window Edit Alternatives Table (Figure 2.12). In particular, during the definition of a new project, actions may be defined one after another. The **Auto Insert Mode** button is the default button.

- **Delete**: deletes the selected action: you will be prompted for confirmation.

- **Help**: gives the user an online help for the contents of the current window.

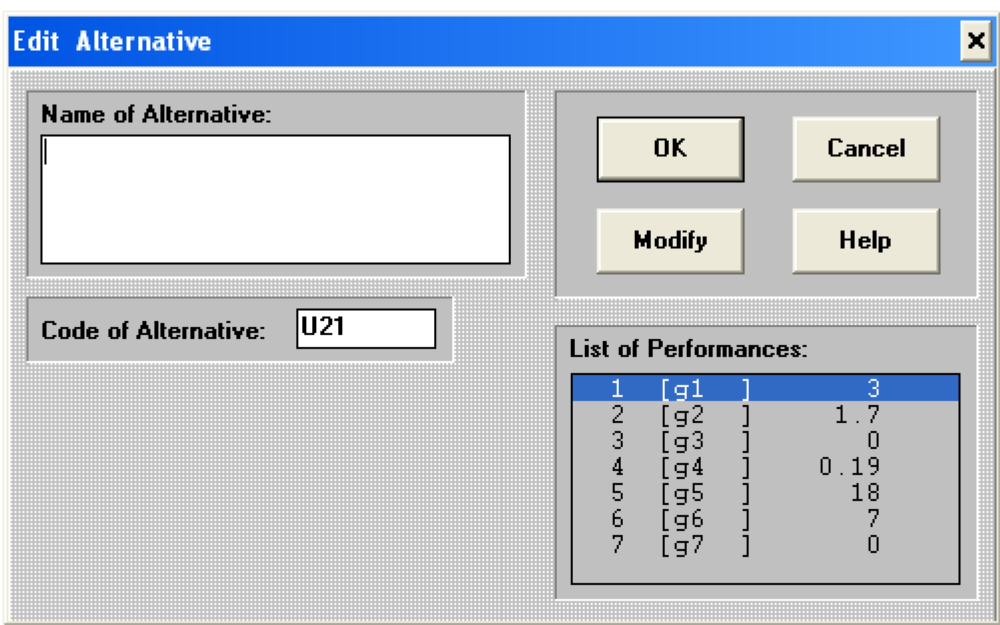


Figure 2.13: Edit alternative

The window Edit Alternative (Figure 2.13) is obtained after the selection of the option **Edit / Alternatives**, followed by the selection of the button **Insert** or the button **Modify**. In the first case (Insert), will show a window where all the entry zones are empty, in the second case (Modify) the zones will contain the characteristics of the action to be modified. An action is defined by a:

1) **Name**: you may enter any string with a maximum of 80 characters in the window, blanks are permitted.

2) **Code**: consisting in five alpha-numerical characters. Any letter or digit is permitted. If you enter no code, by default, the program will give to the action the first available code in the list: *A*0001, *A*0002, *A*0003, ...

If a family of pseudo-criteria has already been defined, a window will appear and display the list of codes for the criteria and the performances of the selected action on all the criteria. It is possible to enter or modify the performance of the selected action on the criteria (Figure 2.14). See above the button **Modify**. The window Edit Alternative (Figure 2.13) has four buttons: OK, Cancel, Modify, and Help:

- **OK**: closes the window Edit Alternative and stores in memory the characteristics entered for this action. The following events depend on the way you entered this option:

  1) if you entered this window by selecting the button **Modify** in the window Edit Alternatives Table (Figure 2.12) or the button **Insert** in the same window with the button **Auto Insert Mode** not activated, activating the button **OK** brings you back to the window Edit Alternatives Table. It is then possible to modify another action, to insert an action in another place of the list or to quit the window Edit Alternatives Table.

  2) if you entered this window by selecting the button **Insert** in the window Edit Alternatives Table (Figure 2.12) with the button **Auto Insert Mode** activated, clicking the button **OK** displays a new window Edit Alternative with empty entry zones: you may define a new action which will be inserted in the list after the current action. If you do not wish to enter the codes for your actions you will define your family of actions very quickly by clicking **OK** (or by pressing the

ENTER key) a number of times corresponding to the number of actions to be created. The codes will be created automatically.

- **Cancel**: closes the window but does not store in memory the possible modifications of the entry zones. This is the best way to stop an automatic insertion of actions or to cancel the modifications that have just been made to the definition of an action.

- **Modify**: allows to modify or to enter the performance of the current action on a criterion. The two following actions are equivalent:

  1) select a criterion in the list by clicking once on its code then click on the button Modify.

  2) double-clik on the code of the criterion in the list of performances.

  A window Edit Performance (Figure 2.14) is displayed and you may modify the performance of the current action on the selected criterion.

- **Help**: gives the user an online help for the contents of the current window.

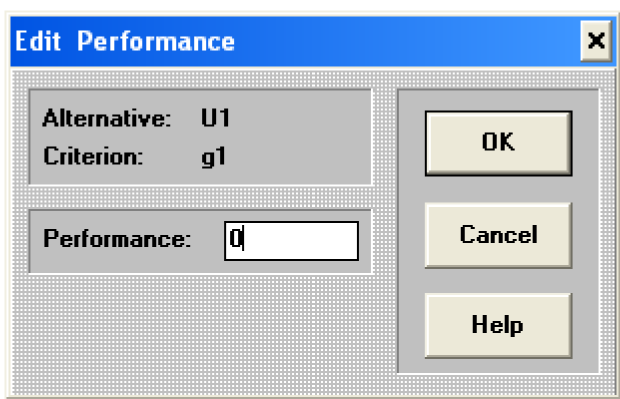
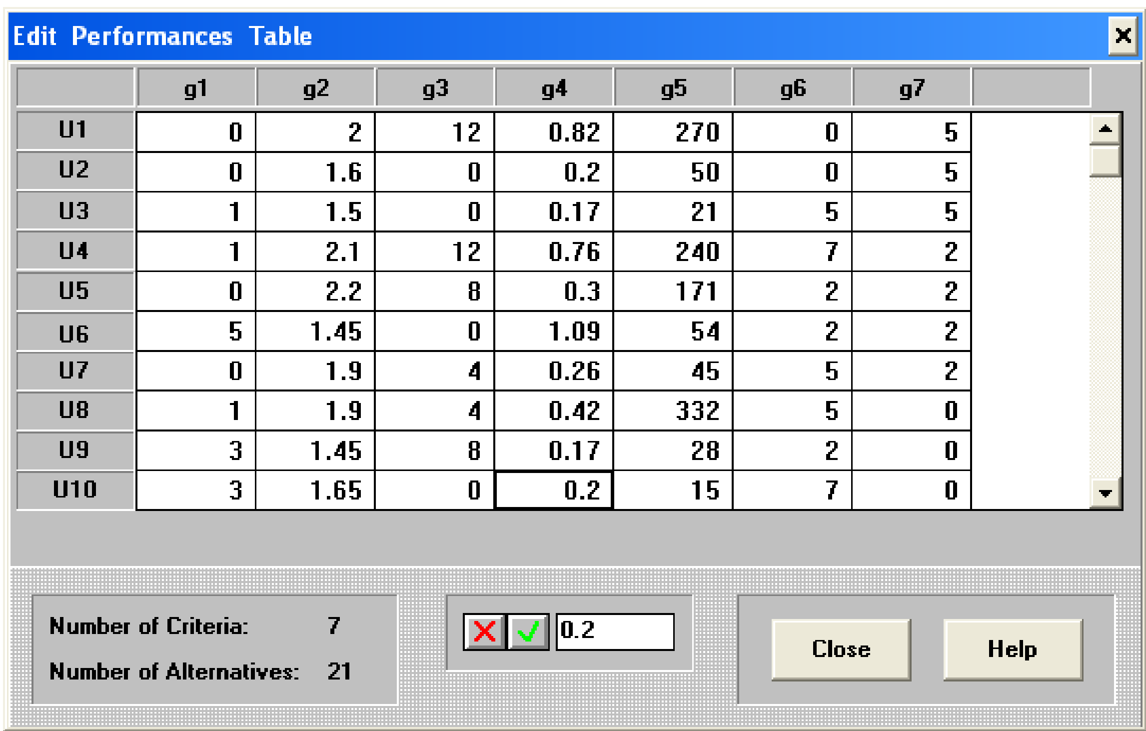


Figure 2.14: Edit performance

The Figure 2.14 is obtained after the selection of the option **Edit / Criteria** or the option **Edit / Alternatives**, followed by the selection of the button **Insert** or the button **Modify**, and then on the button **Modify**. By this way, the performance of a given action on a specific criterion may be defined or modified. It is preferable, during the definition of a new project to enter the performances using the option **Edit / Performances**. For more details, see the next section.

2.2.2.4 Edit Performances

The option **Edit / Performances** allows to enter the performance of each action on each criterion (Figure 2.15). It is possible to enter integers or decimal numbers, positive or negative. The choice of the units on the scale of values is not important: for example, for a cost you can type your values in € or thousands of €. Each performance must be within the limits $-999,999$ and $999,999$. It is possible to type six significant digits (including decimals): 1235436.2 will be rounded to 1235436; 10987.358 will be rounded to 10987.4 and -7749.576 will be rounded to -77459.6.



	g1	g2	g3	g4	g5	g6	g7
U1	0	2	12	0.82	270	0	5
U2	0	1.6	0	0.2	50	0	5
U3	1	1.5	0	0.17	21	5	5
U4	1	2.1	12	0.76	240	7	2
U5	0	2.2	8	0.3	171	2	2
U6	5	1.45	0	1.09	54	2	2
U7	0	1.9	4	0.26	45	5	2
U8	1	1.9	4	0.42	332	5	0
U9	3	1.45	8	0.17	28	2	0
U10	3	1.65	0	0.2	15	7	0

Figure 2.15: Edit performances table

The typing is of a spreadsheet type: one line per action and one column per criteria. The size of this window is fixed and lets at most 10 actions and 8 criteria appear. The other performances are accessible with the scrolling. A box recalls the number of actions and criteria. The actions and the criteria appear under their code name in the order they have been inserted. To enter the performances:

1) move the border to the cell in which you want to enter a number by using the arrow key or by clicking with the mouse in the cell.

2) type the number (as you type the entry in the formula bar, the software displays it in the active cell).

3) confirm the entry in the cell by clicking the check mark in the formula bar or by pressing **ENTER** key. There is a shortcut to confirm the entry in the cell and move the border of the active cell. Instead of pressing **ENTER**, you can just press an arrow key or click with the mouse in another cell. If you make a typing mistake, press the backspace key and retype the number again.

Beware, if you modify a performance on a criterion whose thresholds have already been defined, it may exceptionally happen that the new value invalidates the threshold coefficients: in this case, you will be warned and asked to modify the threshold coefficients to make them consistent with the new value. You may save your project even if the table of performances is not complete yet. The next time you open the project, a dialog box will warn you that all performances have not been typed.

The button **Close** closes the window Edit Performances Table (Figure 2.15) and stores in the memory of the computer the modifications of the performance table. Beware, to save on the disk, you must select the option **File / Save Project**.

2.2.2.5 Edit Thresholds

The option **Edit / Thresholds** allows to define or modify the values of the thresholds for the family of pseudo-criteria (Figure 2.16). This option must be selected only after having define a set of actions, a family of criteria and a table of performances (see the sections 2.2.2.2, 2.2.2.3, 2.2.2.4 on pages 62, 66, 70, respectively, for more details). The total number of criteria is recalled. The list of the codes for the criteria appears in the list of criteria and also complementary information of the selected criterion. This information (preference direction, definition mode, and coefficients of the different thresholds) is updated each time a new criterion is selected. In the bottom of Figure 2.16 there are three buttons: Close, Modify, and Help:

- **Close**: closes the window and stores in the memory of the computer the modifications to the definitions of thresholds. Beware, to save on the disk, you must select the option **File / Save Project**.

- **Modify**: allows to modify the thresholds of a selected criterion. The two following actions are equivalent:

  1) select a criterion by clicking once on its code, then click on the button Modify.
  2) double-click on the code of a criterion in the list of criteria.

  A window Edit Thresholds (Figure 2.17) is opened and you may modify the definition mode of the thresholds and the coefficients of the thresholds function of the criterion.

- **Help**: gives the user an online help for the contents of the current window.

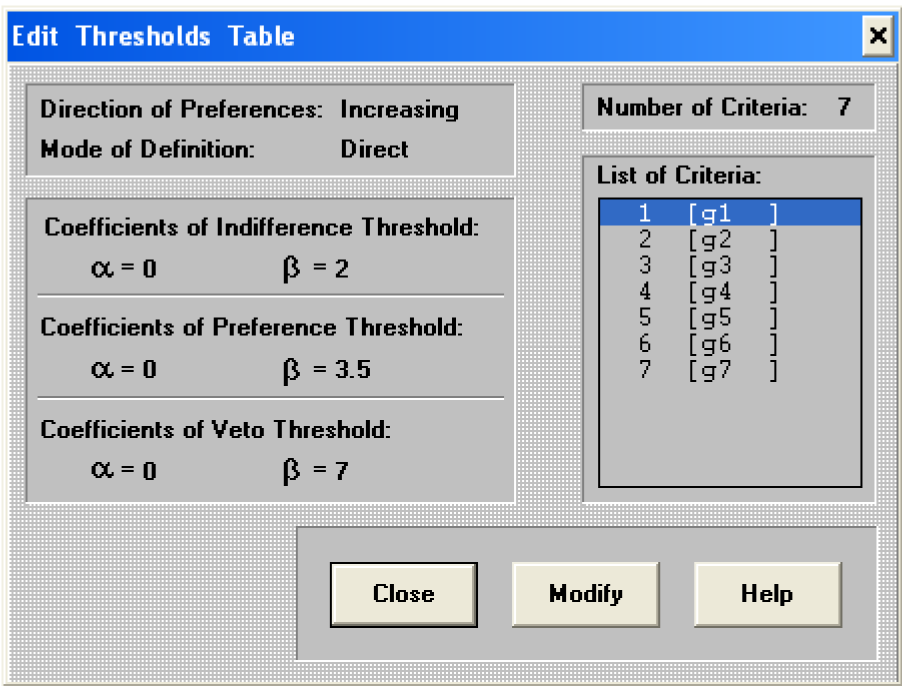


Figure 2.16: Edit thresholds table

A data set of ELECTRE III and/or ELECTRE IV type is complete only if the threshold coefficients have been defined for each criterion. It is therefore necessary to call the window Edit Thresholds (Figure 2.17) for each criterion. This is obtained after the selection of **Edit / Thresholds** and then on the button **Modify** or by double-clicking on the code of a criterion. It allows to define or modify the characteristics of the threshold functions for a specific criterion. In order to be able to define the coefficients of the thresholds for a criterion, it is necessary to have entered the performances of each action on the criterion.

The window Edit Thresholds presented in Figure 2.17 also recalls the code of the selected criterion, its weight and the direction of preference. In order to help the user to build thres-

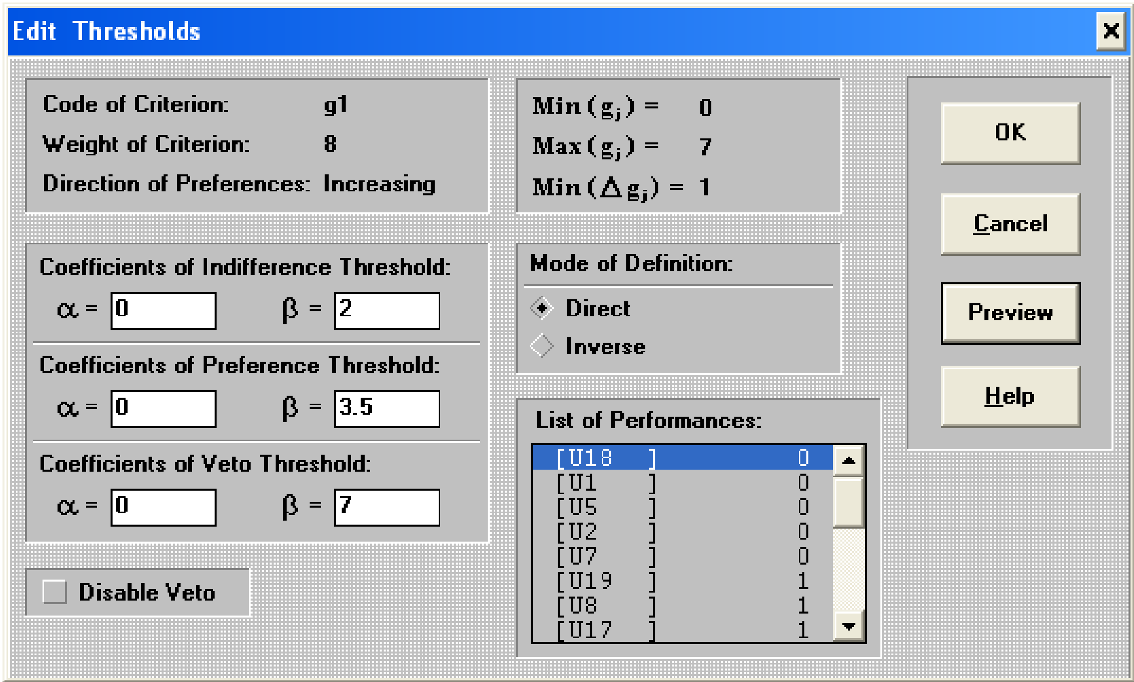


Figure 2.17: Edit thresholds

holds functions it presents the value of the smallest performance [min(g_j)]; the value of the largest one [max(g_j)]; and the smallest non-zero difference between two performances [min$(\Delta\, g_j)$]. This last value may help the user build the indifference threshold. It also recalls the list of the performances of every action on selected criterion (the performances are ranked from the worst to the best). It is advised to begin with values which seem to fit at best in order to define a reference set and then to modify the values reasonably to make a robustness analysis. The compulsory zones to be filled are the following:

- coefficients α and β of the indifference and preference threshold, as an affine function: $\alpha \times g(a) + \beta$.
- either the button **Disable Veto** or the coefficients α and β of the veto threshold, as an affine function: $\alpha \times g(a) + \beta$.
- one of the two options in the **Mode of Definition**: Direct or Inverse.

In the window Edit Thresholds (Figure 2.17) there are three buttons: OK, Cancel, Preview, and Help:

- **OK**: closes the window and stores in memory the modifications of the zones.

- **Cancel**: closes the window without taking into account the modifications that may have happened to the entry zones.

- **Preview**: helps you to determine the coefficients α and β that fit best by showing the values taken by the threshold functions for every performance. This button can also be used to help the user to observe the consequences on the values of the thresholds when the values of the coefficients are modified (Figure 2.18).

- **Help**: gives the user an online help for the contents of the current window.

The Figure 2.18 is obtained after the selection the option **Edit / Thresholds**, the button **Modify** and lastly the button **Preview**. The actions are ranked from the worst to the best: for each action, the software displays the performance, the indifference, and the preference thresholds entered by the user and the veto threshold if any.

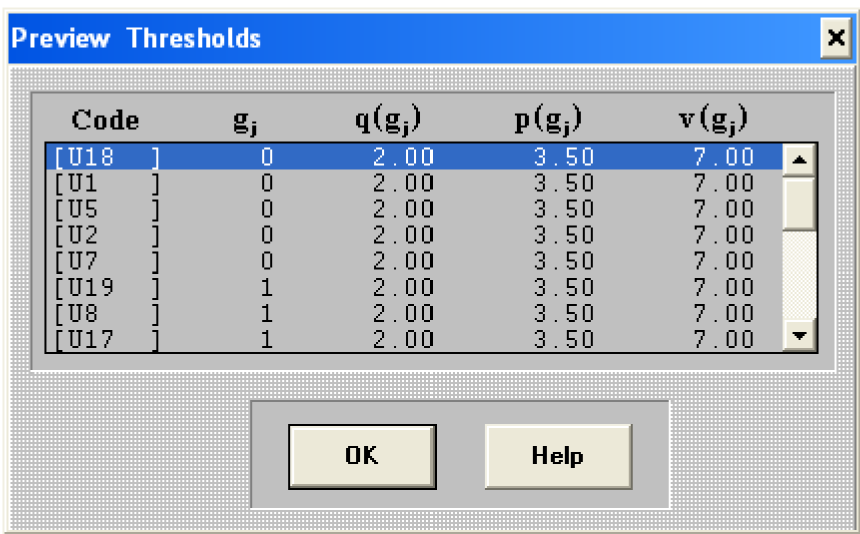


Figure 2.18: Preview thresholds

No defining thresholds for a criterion, constant or proportional thresholds, direct or inverse definition's modes, disable veto, and what are the constraints on the thresholds must be taken into account on the edition of the thresholds (Figure 2.17), as follows:

- **No defining thresholds for a criterion**:

  - If the indifference and preference thresholds for the selected criterion are not relevant (perhaps it is a true-criterion), you should enter $\alpha = 0$ and $\beta = 0$ for the indifference and preference thresholds.

- **Constant or proportional thresholds**:
  - If you wish to define a pseudo-criterion, you should type the appropriate values in order to build the indifference and preference value function.
  - If you wish to define a constant threshold on the whole scale of performances, you only have to type $\alpha = 0$ and $\beta =$ constant value of the threshold. In this case, the two definition modes, Direct or Inverse, are equivalent.
  - To define a threshold (direct or inverse) proportional to the performance, type $\alpha \neq 0$.
- **Direct or Inverse definition modes**:
  - The indifference, preference, and veto thresholds may be calculated with the best or the worst performance of a and b: in the first case the calculation of the thresholds is said to be Direct and Inverse in the second case.
  - For example, let us consider a criterion for which a 20% indifference threshold has been defined. If the definition mode of the threshold for the criterion is direct, two actions with performances respectively 80 and 100 on this criterion will not be indifferent (indifference threshold $= 16$), but if the definition mode is inverse these two actions will be considered as indifferent (indifference threshold $= 20$).
  - In order to apply always the same ranking algorithm, the software computes itself automatically the necessary calculations to convert inverse thresholds into direct ones. If unspecified, the software activates the Direct option on the mode of definition.
- **Disable Veto**
  - The role of the veto threshold is completely different from the ones of the indifference and preference thresholds. The last two are internal parameters for the criterion under consideration (intra-criterion parameters) whereas giving a criterion the opportunity to veto modifies its role (importance) as compared to the other criteria, veto threshold coefficients are inter-criteria parameters.
  - It is possible to give the opportunity to veto to one, several or every criterion. In this case, you should not activate the button **Disable Veto** and define the coefficients α and β of the veto threshold functions.

- **What are the constraints on the thresholds?**

  - The coefficients α and β of the threshold functions should verify several conditions. Checking is made when these coefficients are entered. There are three types of constraints:

    1) no threshold may be negative, that is the number $\alpha \times g(a) + \beta$ must be positive for all $g(a)$.

    2) the threshold functions may be increasing or decreasing but for consistency reasons, the proportionality coefficient should verify a few constraints:

       . it must be greater then -1 in the case of increasing preference direction and direct threshold and in the case of decreasing preference direction and inverse thresholds;

       . it must be smaller than 1 in the case if increasing preference direction and inverse threshold and in the case of decreasing preference direction and direct thresholds.

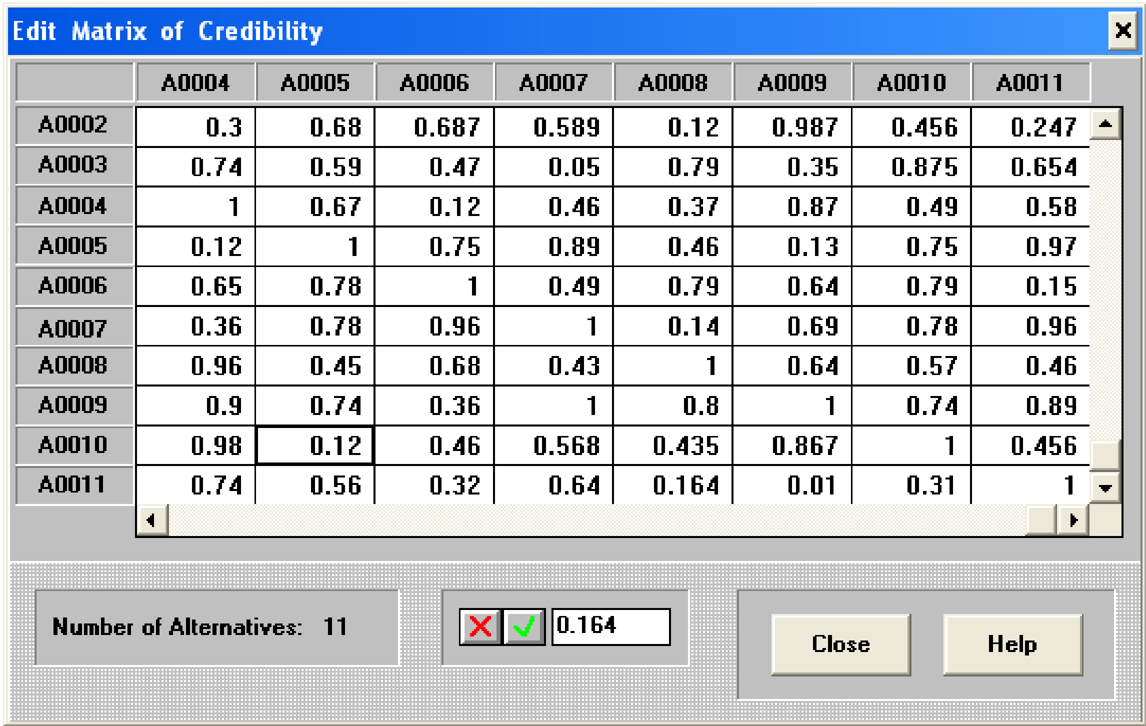
    3) the indifference threshold should stay smaller than or equal to the preference threshold which should, in turn, stay smaller than or equal to the veto if there is one.

  - If one of these constraints is violated, the software will display a window asking you to modify the inappropriate values.

2.2.2.6 Edit Matrix of Credibility

The option **Edit / Matrix of Credibility** is available only for the projects which have the *Matrix of degrees of credibility* type (Figure 2.19). It is possible to apply the ranking algorithm of ELECTRE III method to any matrix of pairwise comparisons of actions even if it was obtained with rules different from those of ELECTRE III or ELECTRE IV methods. It is necessary to have defined beforehand all the actions before accessing this option. This window recalls the total number of actions.

The typing is of spreadsheet form: the list of the codes for the actions appears on the rows and the columns. At the intersection of row $A0003$ and column $A0001$, for instance, you may enter the value of the credibility degree of $A0003$ over $A0001$, that is the index which expresses to what extent $A0003$ is at least as good as $A0001$. This index is a decimal number



	A0004	A0005	A0006	A0007	A0008	A0009	A0010	A0011
A0002	0.3	0.68	0.687	0.589	0.12	0.987	0.456	0.247
A0003	0.74	0.59	0.47	0.05	0.79	0.35	0.875	0.654
A0004	1	0.67	0.12	0.46	0.37	0.87	0.49	0.58
A0005	0.12	1	0.75	0.89	0.46	0.13	0.75	0.97
A0006	0.65	0.78	1	0.49	0.79	0.64	0.79	0.15
A0007	0.36	0.78	0.96	1	0.14	0.69	0.78	0.96
A0008	0.96	0.45	0.68	0.43	1	0.64	0.57	0.46
A0009	0.9	0.74	0.36	1	0.8	1	0.74	0.89
A0010	0.98	0.12	0.46	0.568	0.435	0.867	1	0.456
A0011	0.74	0.56	0.32	0.64	0.164	0.01	0.31	1

Figure 2.19: Edit Matrix of Credibility

between 0 and 1. The values on the diagonal are equal to 1 (an action is clearly at least as good as itself): it is not possible to modify these values.

The size of a window is fixed and lets appear at most 10 actions on the rows and 8 on the columns. The other credibility degrees are accessible by scrolling. The actions appear with their code names in the order they have been inserted. To enter the indices:

1) move the border to the cell in which you want to enter a number by using the arrow key or by clicking with the mouse in the cell.

2) type the number (as you type the entry in the formula bar, the software displays it in the active cell).

3) confirm the entry in the cell by clicking the check mark in the formula bar or by pressing the **ENTER** key. There is a shortcut to confirm the entry in the cell and move the border of the active cell. Instead of pressing enter, you can just press an arrow key or click with the mouse in another cell. If you make a typing mistake, press the backspace key and retype the number

2.2.3 Calculate menu

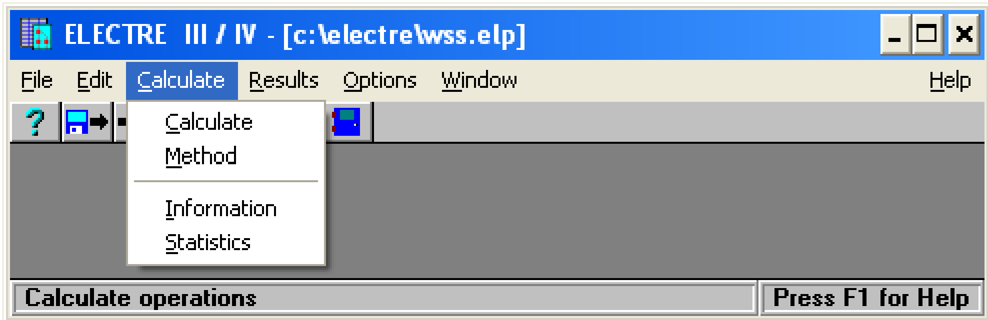


Figure 2.20: Calculate menu

In the **Calculate** menu are available four options - Calculate, Method, Information, and Statistics (Figure 2.20):

- **Calculate** [**ALT+C+C**]: launches the execution of the calculations on the current project (or click on the fifth button in the toolbar from the left to the right). If the project is of both ELECTRE III and ELECTRE IV types, the method should have been chosen using the **Calculate / Method** option.

- **Method** [**ALT+C+M**]: allows to choose the method that has to be used by the **Calculate** option (ELECTRE III or ELECTRE IV) if the project is of both types. It also allows to choose the relations to be used if the selected method is ELECTRE IV, to modify the discrimination coefficients if the selected method is ELECTRE III (or if the project is of *Matrix of degrees of credibility* type).

- **Information** [**ALT+C+I**]: displays information about the current project (directory and current project, number of actions and criteria, project status, and available memory). If the status of the project is <Incomplete> then you cannot make the calculations.

- **Statistics** [**ALT+C+S**]: displays information about the results of the **Calculate** options: used method, relations chosen (ELECTRE IV) or discrimination coefficients (ELECTRE III), number of ranks obtained in each distillation in the final pre-order, in the median pre-order.

2.2.4 Results menu

In the **Results** menu the following options are available (Figure 2.21) - Distillations Result, Ranks in Final Preorder, Median Preorder, Additional Results, and Final Graph:

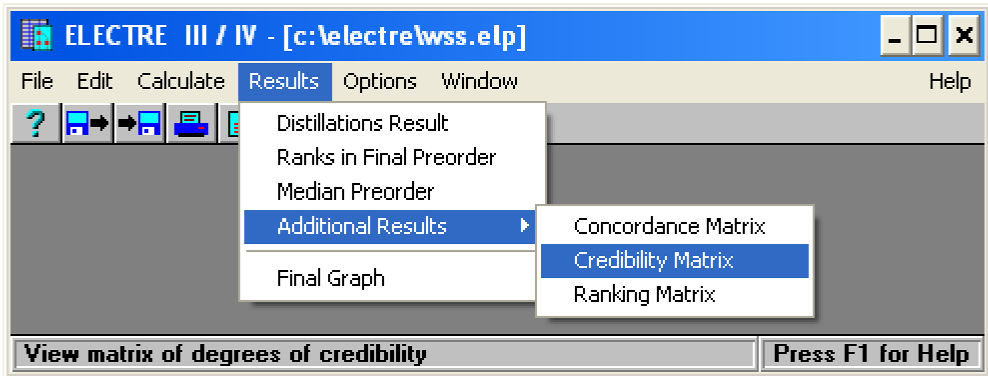


Figure 2.21: Results menu

- **Distillations Result** [ALT+R+D]: displays the results of the two distillations (ascending and descending) for the current project. The software processes the calculations each time this action is launched except if the **Calculate / Calculate** has been previously launched.

- **Ranks in Final Preorder** [ALT+R+F]: displays the list of the actions in each rank of the final pre-order obtained as the intersection of the complete pre-orders from the two distillations. An action has rank n if the better ranked actions have rank $1, 2, \ldots,$ or $n-1$. This option allows to visualize the rank of each action; the incomparabilities are not displayed.

- **Median Preorder** [ALT+R+M]: displays the actions ranked with a median pre-order. This pre-order is a complete one built in the following manner: the actions are ranked following the ranks in the final (partial) pre-order and two incomparable actions in a same rank are ranked according to the differences in their positions in the two distillations.

- **Additional Results** [ALT+R+A]: displays a sub-menu proposing three types of complementary results: another way to visualize the final partial pre-order (using a matrix: Ranking Matrix), results from intermediate calculations such as the Concordance Matrix (only for projects executed with ELECTRE III) or such as the Credibility Matrix.

- **Final Graph** [ALT+R+G]: displays the graph representing the partial pre-order obtained using the ranking algorithm of ELECTRE software.

2.2.5 Options menu

In the **Options** menu the following options are available (Figure 2.22) - Show Toolbar, Show Statusbar, Results, Preferences, and Save Options:

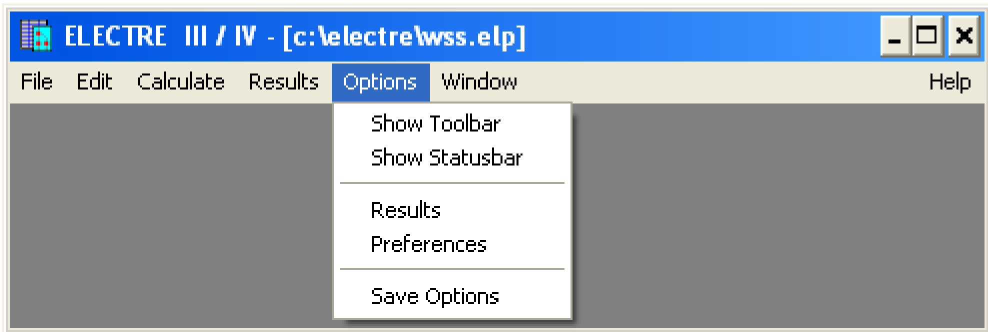


Figure 2.22: Options menu

- **Show Toolbar** [ALT+O+T]: to display or hide the *toolbar*. This bar appears under the main menu and proposes several short-cut buttons for the most often used options.
- **Show Statusbar** [ALT+O+S]: to display or hide the *status bar*. This bar appears at the bottom of the screen and displays short messages on the selected option.
- **Results** [ALT+O+R]: to choose some options about the results. It is possible to define the precision with which the concordance matrix, credibility matrix should be displayed (2 or 3 digits), to choose the symbol set with which the final pre-order (ranking matrix) should be displayed and to activate or not the optimization of the graph (Figure 2.23).
- **Preferences** [ALT+O+P]: to activate the automatic saving of the current options of the option **Options / Results** on one side, and of the size and position of the software desktop window on the other side, at the end of software sessions. If the corresponding option is activated at the end of the software session, the **Results** options of the current project become the default options and the size and position of the software desktop window are saved to be used for next sessions.

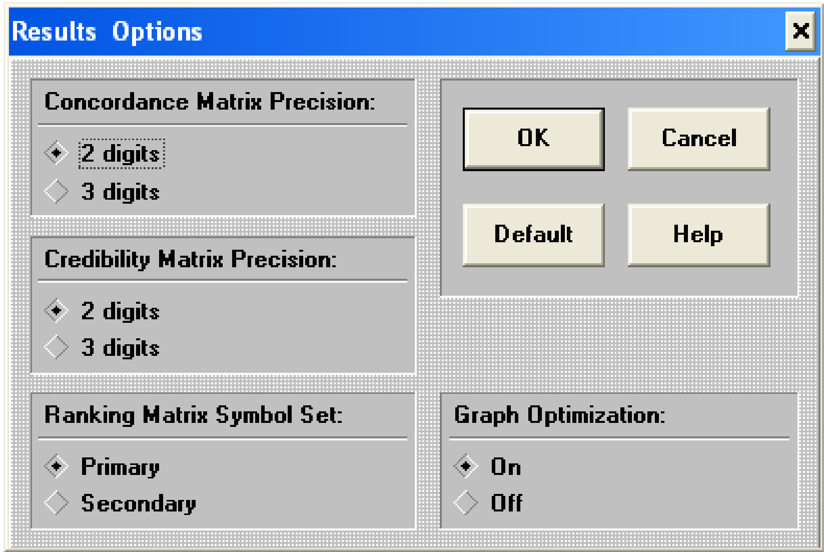


Figure 2.23: Results options

- **Save Options** [ALT+O+O]: to save current options for the option **Results** (i.e. to save them as the default options) and/or to save the size and position of the software desktop window.

2.2.6 Window menu

In the **Window** menu the following options are available (Figure 2.24) - Cascade, Tile, Arrange Icons, and Close All:

- **Cascade** [ALT+W+C]: organizes the different open windows on the desktop in such a way that their titles are always visible despite overlapping.
- **Tile** [ALT+W+T]: organizes the different open windows on the desktop in such a way that they are all entirely visible.
- **Arrange Icons** [ALT+W+I]: reorganizes the different icons on the desktop, they will be gathered at the bottom of the screen.
- **Close All** [ALT+W+A]: closes all windows and icons open on the desktop.

As displayed in Figure 2.24 it is also possible to see all the results windows that are already open for analysis, obtaining by the **Results** menu.

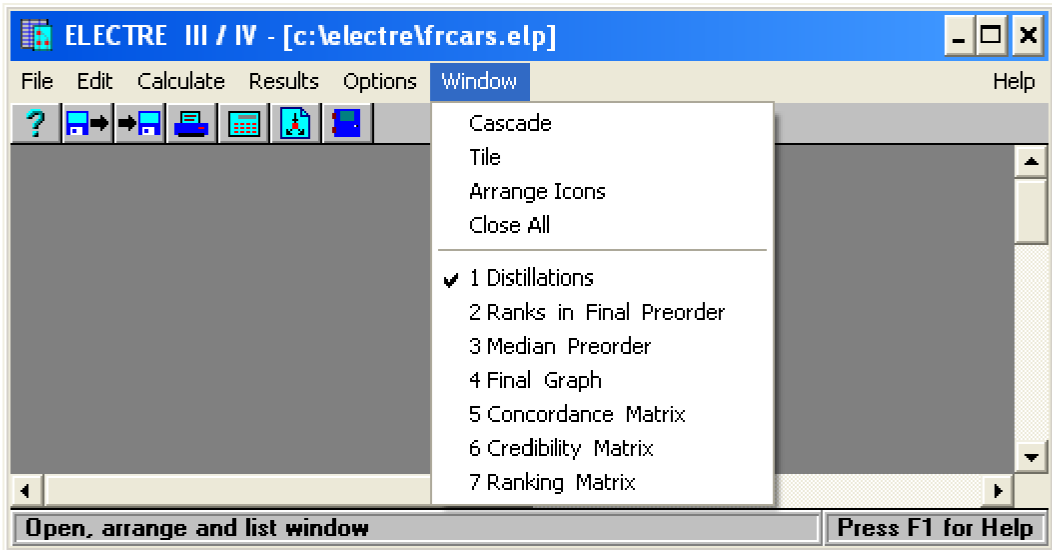


Figure 2.24: Window menu

2.2.7 Help menu

In the **Help** menu the following options are available:

- **Contents** [ALT+H+C]: displays a short help text on every available options in the software.
- **Search Topic** [ALT+H+S]: helps to quickly find the main procedures such as create, execute, read a project...
- **How to Use Help** [ALT+H+H]: displays a help information about using the Windows Help.
- **About** [ALT+H+A]: displays the version number and the copyright note.

2.3 The ELECTRE III calculations

ELECTRE III method uses a set of weights to aggregate partial preferences on each criterion in a fuzzy comprehensive outranking relation. To be able to use it, with this software, you must choose in the **Project Reference** the ELECTRE III project type (Figure 2.9, on page 62). Then the required input data must be entered by using the **Edit** menu (See section 2.2.2 on page 60, for more details). After the input data are completed, it is necessary to choose **Calculate / Method** [ALT+C+M] and on the Current Method section, the ELECTRE III option, which will be used for calculations. This is the default active option (see Figure 2.25). In this window there are four buttons - OK, Cancel, Advanced, and Help:

- **OK**: is used to close the window Method and to store in the memory of the computer the modifications in the project. To save all the options, that have been selected, on the disk it is up to go **File / Save Project**.
- **Cancel**: is used to close the window Method without any modification to the current project.
- **Advanced**: allows to modify the coefficients of the discrimination threshold function used in the ranking algorithm of ELECTRE III. This button is only activated for projects which have either ELECTRE III, or *Matrix of degrees of credibility* type.
- **Help**: gives the user an online help for the contents of the current window.

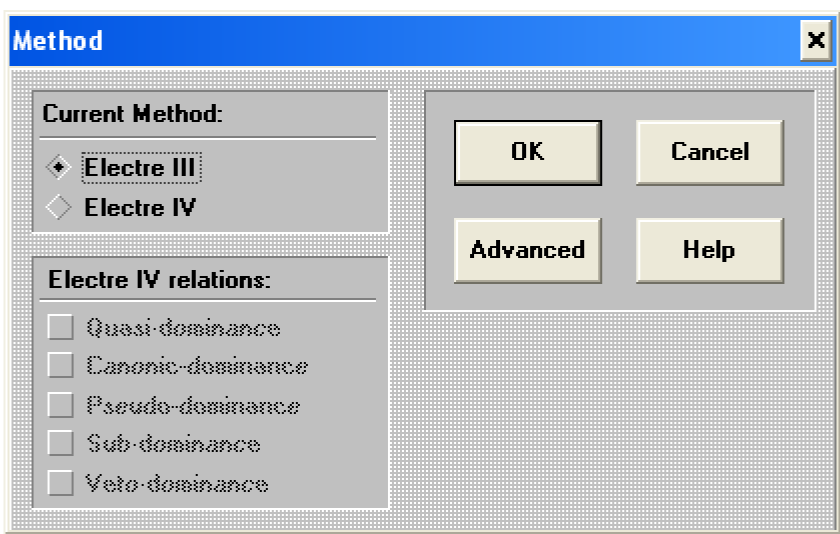


Figure 2.25: Choosing ELECTRE III method

The Figure 2.26 is obtained by the options **Calculate / Method** and then by clicking on the button **Advanced** [ALT+C+M+A]. This option is available only for ELECTRE III or *Matrix of degrees of credibility* type project. It may be used to modify the coefficients of the distillation threshold function. This function, $s(\lambda) = \alpha \times \lambda + \beta$, may be used during distillations to make successive cuts from the fuzzy outranking relation. It allows to define how the cutting levels will evolve during the distillation process. The coefficients must verify several conditions, such as α and β must not return a negative value for a threshold. For more details, see "consistency reasons" on page 6.

The default coefficients are $\alpha = -0.15$ and $\beta = 0.3$. These values should be modified only by the one that have clear understanding on the distillation process used in the ELECTRE III method. It is always possible to come back to the default values by activating the button **Default**. In the window Advanced Options (Figure 2.26) there are three more buttons - OK, Cancel, and Help:

- **OK**: closes the window and stores in memory the values of the distillation coefficients that may have been modified. If afterwards, you select the option **File / Save Project**, these coefficients will be saved with the project.
- **Cancel**: may be used to close the window without taking into account the modifications.
- **Help**: gives the user an online help for the contents of the current window.

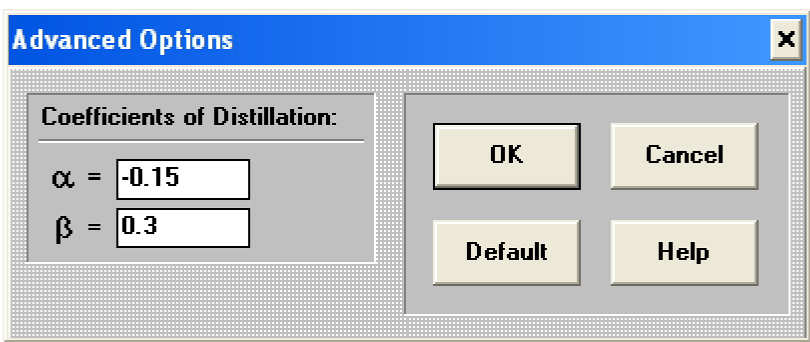


Figure 2.26: Distillations coefficients of ELECTRE III method

The option **Calculate / Calculate** [ALT+C+C] is used to make the calculations for the current project under consideration and to store the results in memory. If you try to make the calculation with an *incomplete project*, a message will warn you and ask you to type the performances of all actions on each criterion and/or the coefficients of the thresholds on each criterion.

If the project is of two types (ELECTRE III and ELECTRE IV), the calculations are executed with the selected method in the **Calculate / Method** [ALT+C+M] option. During the calculation process (for example, in the case study presented in [52]), the software shows a window, like Figure 2.27, that indicates the current status in the calculations, and some statistics related to the number of actions processed, the number of ranks obtained in the descending distillation, the number of ranks obtained in the ascending distillation, the number of ranks obtained in the final pre-order, and the number of ranks obtained for the median pre-order. The user may stop the calculations any time.

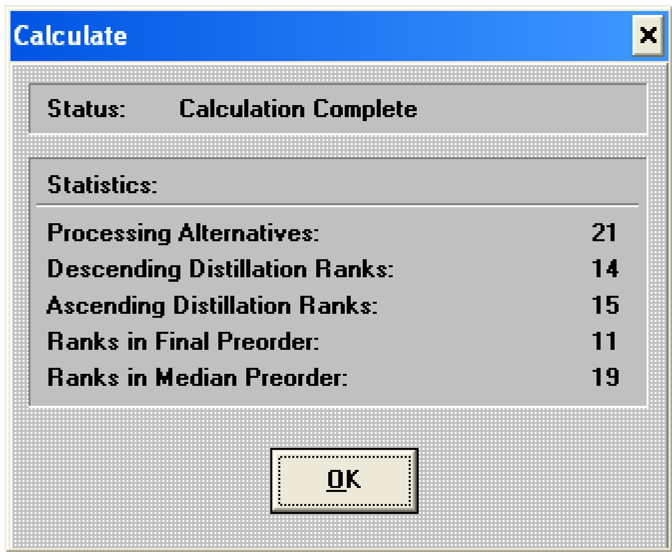


Figure 2.27: Calculation's information

After the option **Calculate / Calculate** [ALT+C+C] has been executed, the **Calculate / Statistics** [ALT+C+S] is available. Clicking on it, the window Statistics (Figure 2.28) recalls some information about the results obtained for the current project (the button **OK** is used to close the window):

- the method used for the calculations (ELECTRE III).
- the distillation coefficients (by default $\alpha = -0.15$ and $\beta = 0.3$ for ELECTRE III. These values can be modified: Figure 2.26, page 84).
- the number of ranks obtained in the descending distillation, the number of ranks obtained in the ascending distillation, the number of ranks of the final pre-order, the number of ranks of the median pre-order.

After that, you have access to the results available for your type of projects in the **Results** menu. You may consult the results for the current project without using this option: though, in this case the calculations are executed each time you select an option in the **Results** menu.

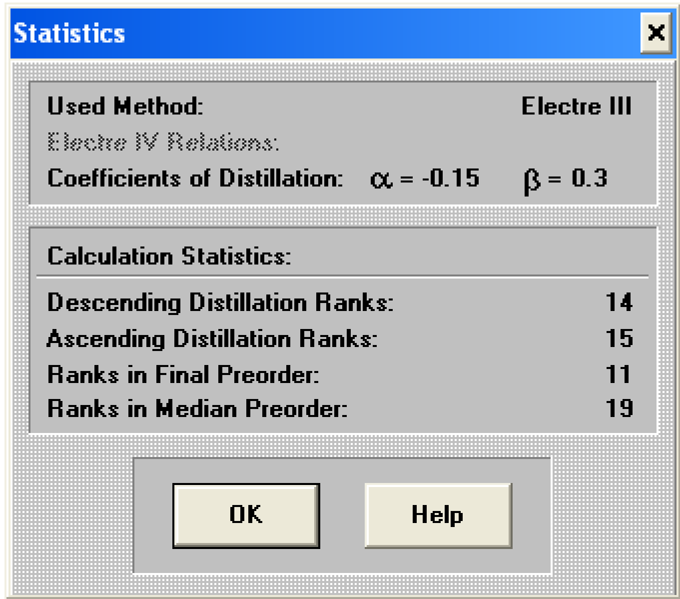


Figure 2.28: Statistics of the ELECTRE III calculation

To speed up the consultations (especially for large projects), you may use once this option, even after any modification of the data.

The option **Results / Distillations** [**ALT+R+D**] must be used to display the results of the two distillations with two contiguous lists (Figure 2.29), after the calculation, but can also make the calculation itself if the **Calculate / Calculate** [**ALT+C+C**] option has not been launched or if some modifications have been made on the data in any case.

The ranking algorithm of ELECTRE III uses the credibility matrix (Figure 2.33) to build two rankings using descending and ascending distillation: descending distillation selects at first the best actions to end the process with the worst ones. On the contrary the ascending distillation selects first the worst actions to end the process with the best ones. Two complete pre-orders are therefore found on all the actions. An action which is incomparable to a group of others will be ranked at the end of this group in the descending distillation and at the top in the ascending distillation. The actions which are considered equal (equivalence classes) in a distillation are displayed in the same box.

In the option **Results / Ranks in Final Preorder** [**ALT+R+F**], only the ranks of the actions in the final pre-order are displayed (Figure 2.30). For a complete representation of this pre-order you may use the options **Results / Final Graph** [**ALT+R+G**] or **Results / Additional Results / Ranking Matrix** [**ALT+R+A+R**].

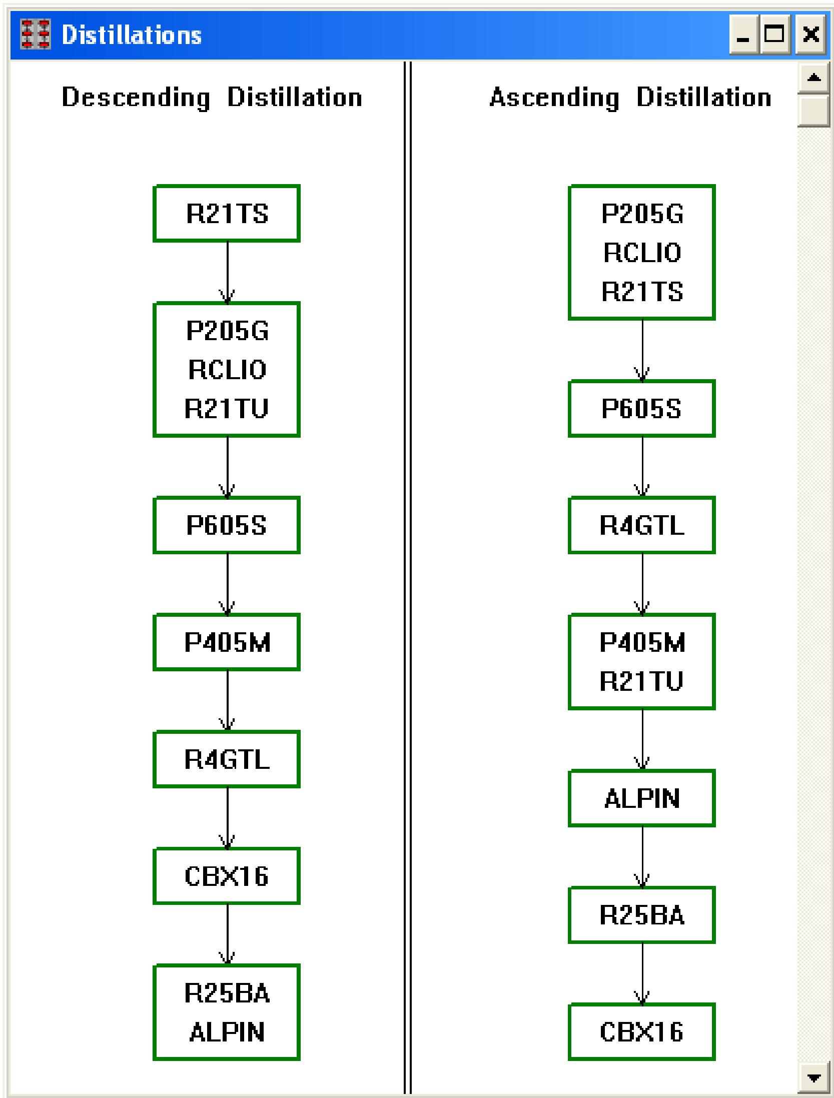


Figure 2.29: Distillations results in ELECTRE III

After the two distillations, the software makes two pre-orders on all the actions. In order to highlight the possible incomparabilities between actions, the method builds the pre-order which is the intersection of the two distillations results:

- an action *a* will be considered as better than an action *b*, if in at least one of the rankings, *a* is ranked before *b*, and if in the other *a* is at least as good as *b*.
- an action *a* will be considered equivalent to an action *b* if the two actions belong to the same equivalence class in the two pre-orders.
- the actions *a* and *b* are incomparable if *a* is better ranked than *b* in ascending distillation and *b* is better ranked than *a* in descending distillation or vice-versa.

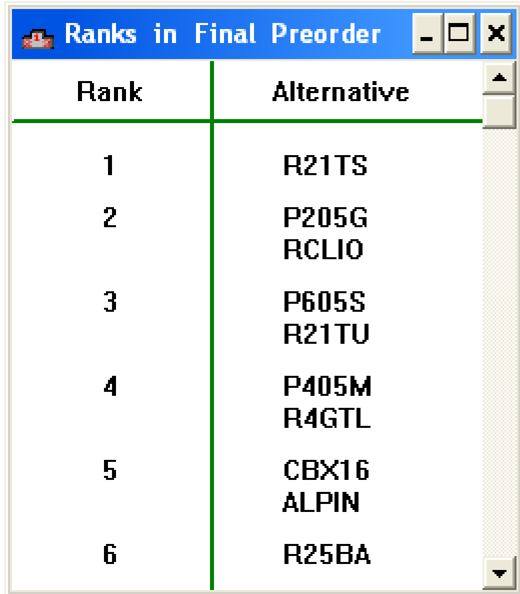


Figure 2.30: Final ranks in ELECTRE III

As a conclusion, in the final pre-order, the comparison between two actions can lead to four different cases:

- *a* may be better than *b*.
- *b* may be better than *a*.
- *a* and *b* may be equivalent.
- *a* and *b* may be incomparable.

The ranks of the actions are built in the following manner: any action which has no better action will have rank 1 (even if it is incomparable to many others), the actions of ranks 2 are those whose better actions are only of rank 1, the actions of rank 3 are those whose better actions are only of rank 1 or 2 and so on... The completely representation hides incomparabilities: two actions with the same ranks may either be equivalent or incomparable and there is no way to discriminate between these two possibilities. In the same way for actions with different ranks, it is not possible to know if an action is better than another one or if they are incomparable. These different cases are clearly distinguished in the display of the matrix of the final pre-order or in the graph.

The option **Results / Median Preorder** [ALT+R+M] (Figure 2.31) shows a ranking which is another possibility for the users who do not wish to take into account incomparabilities.

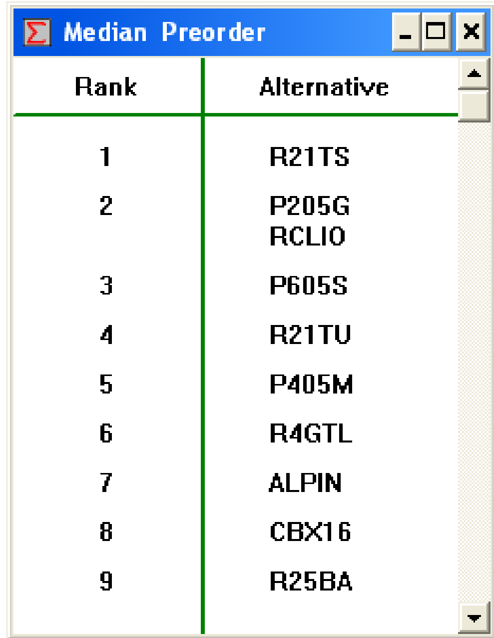


Figure 2.31: Median pre-order in ELECTRE III

This ranking gives a complete pre-order (i.e. any two actions are comparable: one being better than the other or the two being equivalent) built from the final partial pre-order in the following manner: the actions are ranked following the ranks in the final pre-order then the actions having the same rank are distinguished according to their rank difference in the two pre-orders (it indicates the relative stability between the two pre-orders).

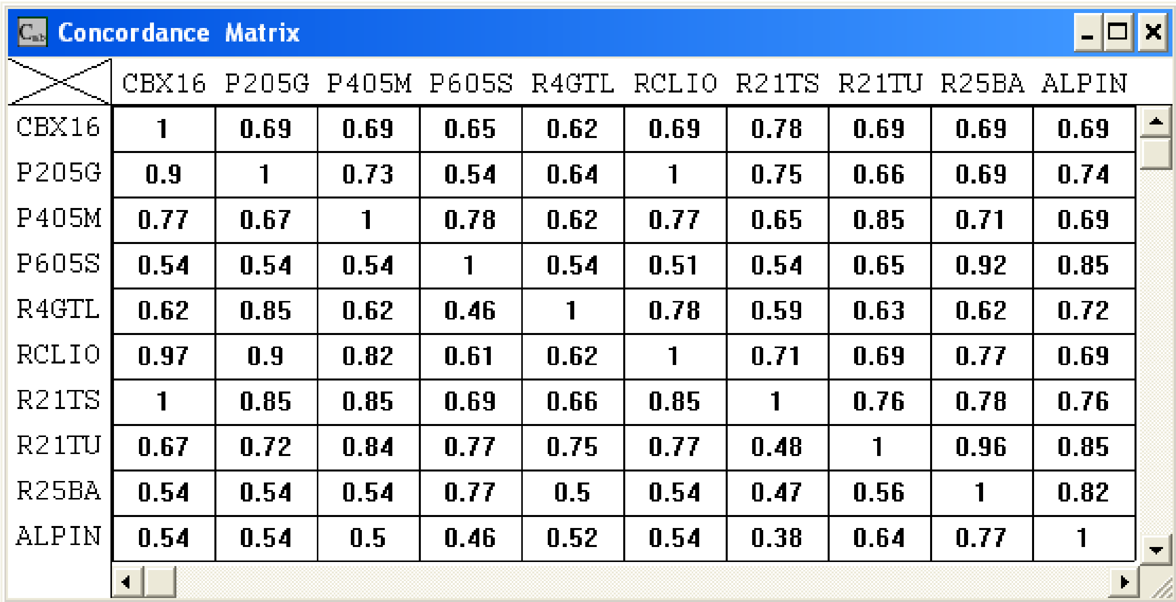
The options of the **Results / Additional Results** [ALT+R+A] will be of any use only for users wishing to improve their knowlarc on the working of ELECTRE methods.

The option **Results / Additional Results / Concordance Matrix** [ALT+R+A+C] displays all the comprehensive concordance indices for every pair of actions (a,b), which is only available for ELECTRE III type project. The comprehensive concordance index $C(a,b)$ is the sum of the concordance indices $C_j(a,b)$ on each criterion weighted by the weights of each criterion, w_j. $C_j(a,b)$ is calculated from the comparison of actions a and b on criterion g_j:

- if the performance of a is greater than or equal to the one of b or if the performance of a is smaller than the one of b but a staying indifferent to b then $C_j(a,b) = 1$.

- if b is weakly preferred to a: $C_j(a,b)$ is obtained with an linear interpolation and is between 0 and 1.

- if b is strictly preferred to a then $C_j(a,b) = 0$.



	CBX16	P205G	P405M	P605S	R4GTL	RCLIO	R21TS	R21TU	R25BA	ALPIN
CBX16	1	0.69	0.69	0.65	0.62	0.69	0.78	0.69	0.69	0.69
P205G	0.9	1	0.73	0.54	0.64	1	0.75	0.66	0.69	0.74
P405M	0.77	0.67	1	0.78	0.62	0.77	0.65	0.85	0.71	0.69
P605S	0.54	0.54	0.54	1	0.54	0.51	0.54	0.65	0.92	0.85
R4GTL	0.62	0.85	0.62	0.46	1	0.78	0.59	0.63	0.62	0.72
RCLIO	0.97	0.9	0.82	0.61	0.62	1	0.71	0.69	0.77	0.69
R21TS	1	0.85	0.85	0.69	0.66	0.85	1	0.76	0.78	0.76
R21TU	0.67	0.72	0.84	0.77	0.75	0.77	0.48	1	0.96	0.85
R25BA	0.54	0.54	0.54	0.77	0.5	0.54	0.47	0.56	1	0.82
ALPIN	0.54	0.54	0.5	0.46	0.52	0.54	0.38	0.64	0.77	1

Figure 2.32: Matrix of concordance in ELECTRE III

The option **Results / Additional Results / Credibility Matrix** [**ALT+R+A+M**] is available for ELECTRE III type of project. The Figure 2.33 displays the credibility degrees that are obtained by the concordance indices taking into account the notion of discordance. The credibility degree $\sigma(a,b)$ is calculated by weakening the concordance index $C(a,b)$ with a formula using discordance indices. These indices are calculated using all the discordant criteria (those for which b is preferred to a) and the values of the veto thresholds.

In the case of a *Matrix of degrees of credibility* type project, the matrix displayed will be identical to the one entered in the option **Edit / Matrix of Credibility** except the fact that the values will be rounded according to the precision defined in the option **Options / Results**.

The option **Results / Additional Results / Ranking Matrix** [**ALT+R+A+R**] displays the matrix of the final pre-order (Figures 2.34 and 2.35) which offers a synthetic view of the results of the ranking method just as the option **Results / Final Graph** [**ALT+R+G**]. These are two different representations of the same final partial pre-order.

During the printing, if you select the option **Ranking Matrix**, the printed characters will correspond to the last symbol set that has been selected with the option **Options / Results**.

The option **Results / Final Graph** [**ALT+R+G**] may be used to visualize the final pre-order using a graph (Figure 2.36). In order to lighten the display, transitivity arcs are omitted. If

Credibility Matrix

	CBX16	P205G	P405M	P605S	R4GTL	RCLIO	R21TS	R21TU	R25BA	ALPIN
CBX16	1	0.69	0.69	0	0.15	0.69	0.78	0.38	0.56	0.38
P205G	0.9	1	0.73	0.16	0.62	1	0.75	0.66	0.69	0.74
P405M	0.77	0.67	1	0	0	0.77	0.65	0.85	0.71	0.69
P605S	0	0	0.39	1	0	0	0	0.65	0.92	0.85
R4GTL	0.4	0	0	0	1	0	0.33	0	0	0
RCLIO	0.97	0.9	0.82	0	0.52	1	0.71	0.69	0.77	0.69
R21TS	1	0.85	0.85	0.47	0.65	0.85	1	0.76	0.78	0.76
R21TU	0.67	0.72	0.84	0.77	0	0.77	0.42	1	0.96	0.85
R25BA	0	0	0.32	0.77	0	0	0	0.48	1	0.82
ALPIN	0	0	0	0.35	0	0	0	0	0.77	1

Figure 2.33: Matrix of credibility in ELECTRE III

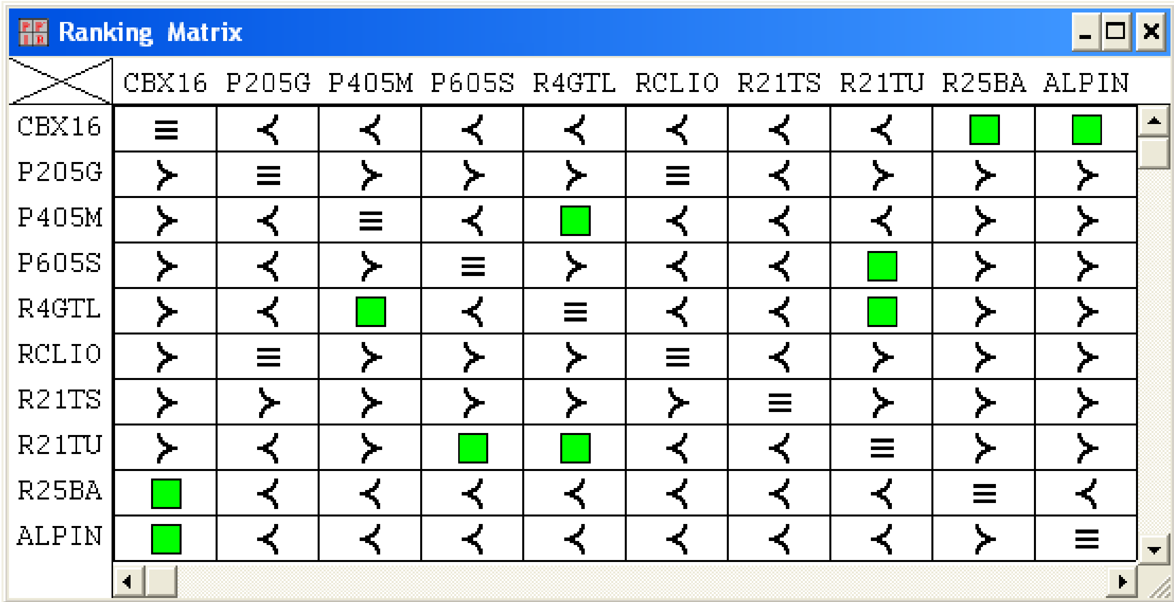
Ranking Matrix

	CBX16	P205G	P405M	P605S	R4GTL	RCLIO	R21TS	R21TU	R25BA	ALPIN
CBX16	I	P^-	P^-	P^-	P^-	P^-	P^-	P^-	R	R
P205G	P	I	P	P	P	I	P^-	P	P	P
P405M	P	P^-	I	P^-	R	P^-	P^-	P^-	P	P
P605S	P	P^-	P	I	P	P^-	P^-	R	P	P
R4GTL	P	P^-	R	P^-	I	P^-	P^-	R	P	P
RCLIO	P	I	P	P	P	I	P^-	P	P	P
R21TS	P	P	P	P	P	P	I	P	P	P
R21TU	P	P^-	P	R	R	P^-	P^-	I	P	P
R25BA	R	P^-	P^-	P^-	P^-	P^-	P^-	P^-	I	P^-
ALPIN	R	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P	I

Figure 2.34: Ranking matrix in ELECTRE III (primary symbol)

there are consecutive arcs from action *a* to action *b*, it is possible to say that *a* is better than *b* in the pre-order obtained. Several equivalent actions are displayed in the same box. Two actions that are not related by an arc or a succession of consecutive arcs are incomparable.

The user may choose between two sets of symbols to represent the relations between actions. It may be chosen in the option **Options / Results**. The actions are ranked in the order they have been initially defined. In order to explain the symbols placed in the Figures 2.34 and


Ranking Matrix

	CBX16	P205G	P405M	P605S	R4GTL	RCLIO	R21TS	R21TU	R25BA	ALPIN
CBX16	≡	≺	≺	≺	≺	≺	≺	≺	□	□
P205G	≻	≡	≻	≻	≻	≡	≺	≻	≻	≻
P405M	≻	≺	≡	≺	□	≺	≺	≺	≻	≻
P605S	≻	≺	≻	≡	≻	≺	≺	□	≻	≻
R4GTL	≻	≺	□	≺	≡	≺	≺	□	≻	≻
RCLIO	≻	≡	≻	≻	≻	≡	≺	≻	≻	≻
R21TS	≻	≻	≻	≻	≻	≻	≡	≻	≻	≻
R21TU	≻	≺	≻	□	□	≺	≺	≡	≻	≻
R25BA	□	≺	≺	≺	≺	≺	≺	≺	≡	≺
ALPIN	□	≺	≺	≺	≺	≺	≺	≺	≻	≡

Figure 2.35: Ranking matrix in ELECTRE III (secondary symbol)

2.35, the pair (a,b) verifies one of the four following relations:

- if *a* is better than *b*, the symbol at the intersection of the row for *a* and the column for *b* is **P** for the primary set of symbol and $\succ$ for the secondary set of symbol.
- if *a* is equivalent to *b*, the symbol **I** or $\equiv$.
- if *a* is ranked worst than *b* in one of the pre-orders and at least as well ranked in the other pre-order, the symbol $\mathbf{P}^-$ or $\prec$.
- if *a* is incomparable to *b*, the symbol **R** or a green square $\square$.

The final graph (Figure 2.36) is built in a way to minimize the number of crossings. Normally, the boxes corresponding to equivalence classes are regularly spaced out and left justified, therefore boxes with different ranks appear one under the next. In this case, for a given rank all the hanging arcs (arcs from actions that are incomparable to any other of the considered rank) are broken and go to the right. Using the example, the action R21TS is better than all the others; the action R21TU is better than the actions P405M, ALPIN, CBX16, and R25BA; the actions P205G and RCLIO are indifferent; the action R21TU is incomparable to the actions P605S and R4GTL.

If you select the option **On** in the dialog box from the option **Options / Results** for the option **Graph Optimization**, the drawing of the graph is optimized in such a way that some

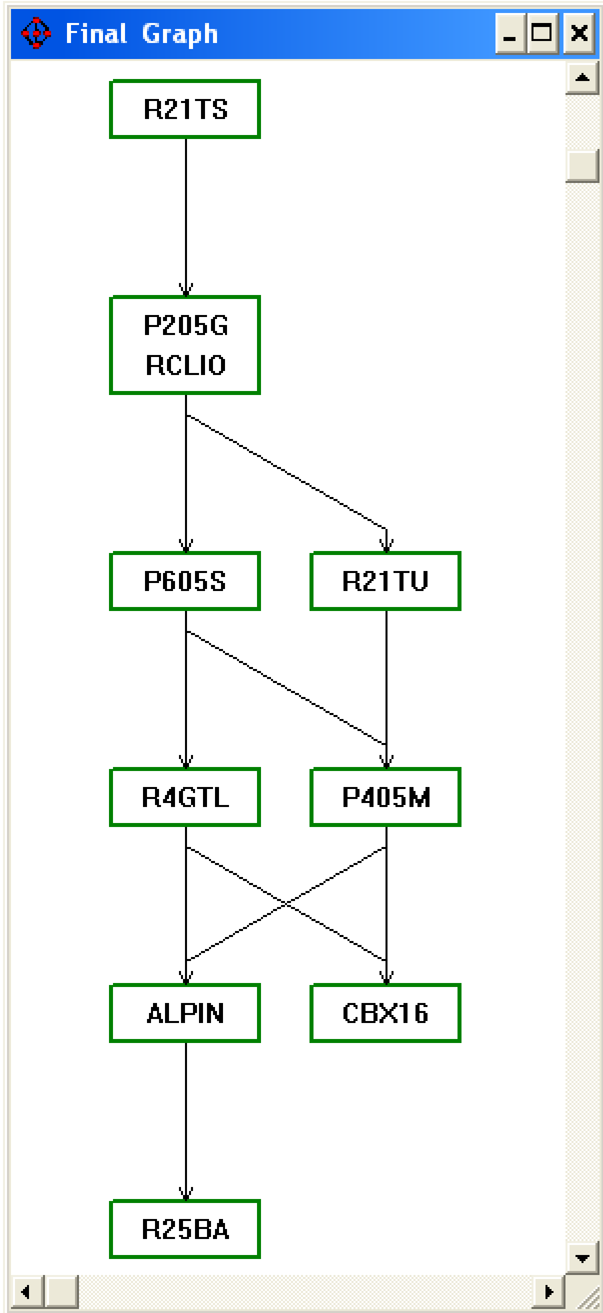


Figure 2.36: Final graph in ELECTRE III

equivalence boxes may be horizontally translated and permuted with others to decrease the number of crossings. The hanging arcs are in this case on the left and on the right. In any case you may choose the representation you like better. The graph may be totally printed whatever its length.

2.4 The ELECTRE IV calculations

ELECTRE IV method may be chosen when you are not able to define *weights* for criteria. For more details see section 1.2 on page 19. To be able to use the ELECTRE IV method, with this software, you must choose in the **Project Reference** the ELECTRE IV project type (Figure 2.9, page 62). Then, the required input data must be entered by using the **Edit** menu (see section 2.2.2 on page 60). After the input data are complete, it is necessary to choose **Calculate / Method** [ALT+C+M] and on the Current Method box, the ELECTRE IV option, which will be used for calculations. After selecting ELECTRE IV (or if the project only contains the ELECTRE IV type), the ELECTRE IV relations box is activated (Figure 2.37).

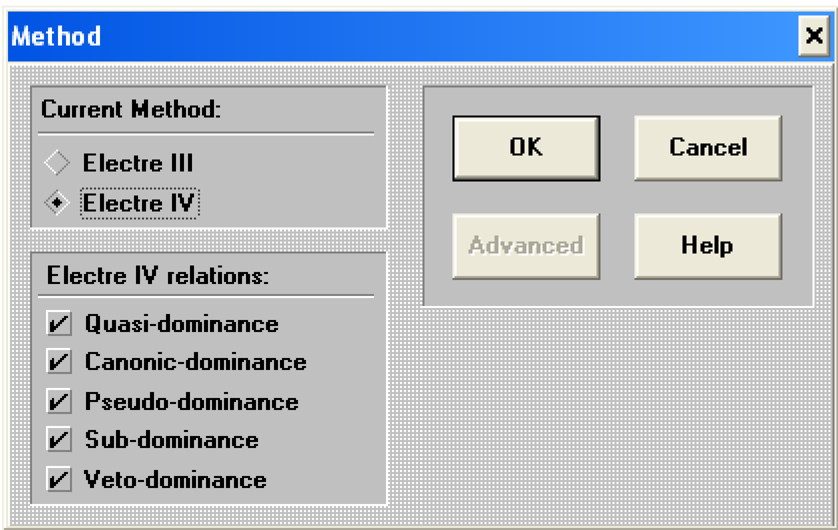


Figure 2.37: Choosing ELECTRE IV method

Now, you may choose among any of the five relations the one you wish to take into account. By default, the software uses the five relations. It is obviously necessary to choose at least one relation. Then, in a way which is totally invisible for the user, the software will associate to each of the selected relations a credibility degree (this value is used for the embedded character of these relations) in order to use the same ranking algorithm than the one of ELECTRE III. After the option **Calculate / Calculate** [ALT+C+C] has been executed the **Calculate / Statistics** [ALT+C+S] is available. Clicking on it, the window Statistics (Figure 2.38) recalls some information about the results obtained for the current project (the button **OK** is used to close the window):

- the method used for the calculations (ELECTRE IV).

- the chosen relations of the ELECTRE IV method: Sq, Sc, Sp, Ss, Sv.

- the distillation coefficients (the values $\alpha = 0$ and $\beta = 0.1$ have been chosen in order to discriminate the outranking relations of ELECTRE IV in successive cuts of the distillation. These values cannot be modified).

- the number of ranks obtained in the descending distillation, the number of ranks obtained in the ascending distillation, the number of ranks of the final pre-order, the number of ranks of the median pre-order.

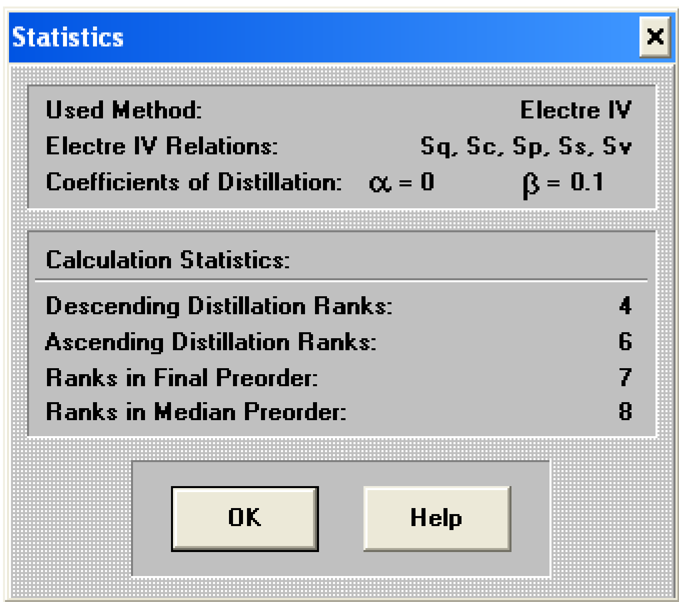


Figure 2.38: Statistics of the ELECTRE IV calculation

The option **Results / Additional Results / Credibility Matrix** [**ALT+R+A+M**] is available for ELECTRE IV type of project. Figure 2.39 displays the credibility degrees that are the values attached to the five relations. For a pair of actions (a,b), only the strongest relation (among those selected by the user) between the two actions will be kept. These values, together with the coefficients of the distillation threshold function, were built in order to be coherent between the distillation mechanism in the ranking algorithm and the successive cuts on the ELECTRE IV relations. Hence:

- the value 1 at the intersection of the row of action a and the column of action b means that the couple (a,b) verifies the relation of *quasi-dominance*.

- the value 0.8 corresponds to the relation of *canonic-dominance*.

- the value 0.6 to the *pseudo-dominance*.

- the value 0.4 to *sub-dominance*.

- the value 0.2 to veto-dominance.

Credibility Matrix

	CBX16	P205G	P405M	P605S	R4GTL	RCLIO	R21TS	R21TU	R25BA	ALPIN
CBX16	1	0	0	0	0	0	0	0	0	0
P205G	0.8	1	0	0	0	1	0	0	0	0
P405M	0	0	1	0	0	0	0	0	0	0
P605S	0	0	0	1	0	0	0	0	0	0
R4GTL	0	0	0	0	1	0	0	0	0	0
RCLIO	0.6	0.4	0	0	0	1	0	0	0	0
R21TS	1	0	0	0	0.2	0	1	0	0	0
R21TU	0	0	0.4	0	0	0	0	1	0.8	0
R25BA	0	0	0	0	0	0	0	0	1	0
ALPIN	0	0	0	0	0	0	0	0	0	1

Figure 2.39: Matrix of credibility in ELECTRE IV

The analysis of the following options by applying the ELECTRE IV method is the same as ELECTRE III method:

- the distillations results, i.e. **Results / Distillations** [ALT+R+D] (like Figure 2.29 on page 87);

- the ranks in the final pre-order, i.e. **Results / Ranks in Final Preorder** [ALT+R+F] (like Figure 2.30 on page 88);

- the median pre-order, i.e. **Results / Median Preorder** [ALT+R+M] (like Figure 2.31 on page 89);

- the ranking matrix, i.e. **Results / Additional Results / Ranking Matrix** [ALT+R+A+R] (like Figures 2.34, and 2.35 on page 91);

- the final graph, i.e. **Results / Final Graph** [ALT+R+G] (like Figure 2.36 on page 93).

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